Workforce Planning at Pizza π*'s:*
How to Minimize Costs and Still Get the Job Done

Linear programming is one of the most powerful mathematical modeling tools in the field of Operations Research. It helps managers find the best ways to allocate limited resources in order to maximize profits or minimize costs (optimization). For example, McDonald's franchises have used linear programming to develop worker schedules which minimize their labor costs. Every *linear programming* model includes:

1.) Decision variables,
2.) An objective function to be maximized or minimized, and
3.) A system of inequalities and equations that represents constraints that restrict the decision maker's options.

Anytime a business or corporation has more than one employee and more than one shift there is a scheduling problem. A *scheduling problem* involves determining the number of employees required to get the work done while minimizing the total daily wages. Many businesses require employees with different skills, but some employees may be able to do more than one job. The process of matching up individuals with tasks is called *manpower planning*. This process becomes more complex when you have to determine a specific work schedule. The most complex manpower planning problem involves the airlines because they have to coordinate flight crews, ground crews, ticket agents and airport staff for flights around the world. Problems of this sort can be solved by a linear programming process.

Sample problem:

A new Pizza π outlet sells pizza for carry out, dining in and frozen pizzas to local groceries and gourmet shops. The Pizza π is open from noon to midnight each day. Toni Pepperoni, the manager of the outlet, needs to figure out how many people should be scheduled to work each of the two 8 hour shifts. The daytime shift is from noon to 8 p.m., and the evening shift is from 4 p.m. to midnight. The two shifts overlap during the busy dinner period, which starts at about 4 p.m. and continues until about 8 p.m. Based on information about the average number of pizzas ordered at other outlets, Toni has estimated the number of employees she needs in each four hour period to supply those pizzas:

noon to 4 p.m.	6 employees
4 p.m. to 8 p.m.	16 employees
8 p.m. to midnight	8 employees

Employees are paid $5 per hour for hours worked between noon and 8 p.m. The pay per hour between 8 p.m. and midnight is $7 per hour. Toni must decide how many people to schedule on the noon to 8 p.m. shift and how many on the 4 p.m. to midnight shift. There must be enough workers on duty to complete the workload in each four hour period and the goal is to minimize the amount spent on daily wages. Answer the following:

1 What decisions must Toni make?____

2 Recommend to Toni the number of workers she should schedule each shift. ______________________________

3 Why did you recommend these numbers?____

____

Completing the questions which follow will determine if your recommendation is a workable solution for Toni. You will also find a solution which minimizes her cost, and will provide enough workers to complete all tasks.

4 Identify all of the variables in this problem. (Hint: read ahead if you get stuck here.)

____

Decision Variables and Constraints:

Two of the variables you should have identified are the number of people working the day shift and the number of people working the evening shift. These are called *decision variables*. All linear programming problems have decision variables. Decision variables should completely describe the decisions to be made. The two decision variables in the Pizza π problem can be represented by:

***Decision variables*:** a set of variable quantities completely describing the decisions to be made

D = number of people scheduled from noon to 8 p.m.; E = number of people scheduled from 4 p.m. to midnight.

5 If three people work the day shift and five people work the evening shift, altogether how much will they make?

____

6 Write an expression to represent the total daily wages paid to employees if Toni schedules D workers for the day shift,

and E workers for the evening shift? Total daily wages = ____

In any linear programming problem the decision maker wants to maximize or minimize some function of the decision variables. This function is called the *objective function*. In the Pizza π problem Toni wants to minimize the total daily wages. Therefore, the *objective function*. is: total daily wages = $40D + 48E$

***Objective Function*:** a quantity to be optimized which is defined in terms of the decision variables

Answering the following questions will help you to understand what is meant by a *constraint*.

Constraints restrictions on the values of one or more of the decision variables

7 Toni must have at least ____ workers scheduled from noon to 4 p.m.; therefore, D _ _______.

8 Toni must have at least ____ workers scheduled from 8 p.m. to midnight, so E _ ________.

9 Write an inequality to show the minimum number of workers needed from 4 p.m. to 8 p.m.:

<u>Hint</u>: Both day and evening shifts are working. ___________________________________.

A System of Inequalities - The Feasible Region:

These inequalities represent restrictions on the decision variables D and E. These restrictions are called *constraints.*

The *optimal solution* is just the best solution to a particular problem. In this case, the values of D and E which satisfy all of the constraints and minimize the total daily wages paid to employees is the *optimal solution*. You may recall that the objective function, total daily wages = $40D + 48E$, defines this cost, where D represents the number of employees working the day shift, and E represents the number of employees working the evening shift. So how *do* we find the optimal solution for this problem? In other words, what is the lowest possible total daily wages that still meets all of the constraints?

First, we need to find all of the values of the decision variables D and E which satisfy all three of the constraints:

$$D \geq 6,\ E \geq 8, \text{ and } D + E \geq 16.$$

> ***Feasible Region***: the set of all points which satisfy all of the constraints

One way to do this is to graph each inequality on the same coordinate system. We need to decide which variable to plot on each axis. Suppose we agree to graph *D* on the horizontal axis and *E* on the vertical axis. On a sheet of your own graph paper, graph each of the three inequalities above and lightly shade the region(s) of the coordinate system containing all of the points satisfying all three inequalities.

The portion of the graph which you shaded is called the *feasible region*. The feasible region contains all of the points which *could* be the optimal solution, because they satisfy all of the constraints.

10 How many points lie in the feasible region? ________________________________

11 Write the coordinates of three points that lie in the feasible region. Include one point that lies on the boundary of the feasible region. (,   ); (,   ); (,   ) Also mark and label the points you choose on your graph.

12 Why are boundary points part of the feasible region? ____

13 Does the point (6,8) lie in the feasible region? ______Why or why not? ________________________________

The Search for Optimality - Finding the Best Schedule:

Now that we know the location of all points satisfying the constraints (the feasible region), we can begin to look for the optimal solution. The optimal solution must be one of the points in the feasible region. But which point? Let's begin by considering a point that we know is feasible. The point (9,10) lies in the feasible region.

14 What does this point represent? ____

If there are 9 day employees and 10 evening employees, then we can use the objective function to find the total daily wages of these employees: Total daily wages = 40 *D* + 48 *E* = 40(9) + 48(10) = 840.

15 Find the total daily wages for each of the three points in the feasible region which you identified in # 11.

D	***E***	**Total daily wages = 40 *D* + 48 *E***

16 Did any of your points produce total daily wages lower than $840? ______________

17 Are there any points in the feasible region that produce total daily wages lower than $840? _______________

18 Which of your points produced the lowest total daily wages? (,   )

19 What were the total daily wages for that point? ____________________

To help find the optimal solution, we will use the graph of the line 40*D* + 48 *E* = 840. Add the graph of this equation to your graph of the feasible region, and label it line *l*. Next, using your answer from #18, add the graph of 40 *D* + 48 *E* = *your* lowest total daily wages to the previous graph and label it line *m*.

20 What do you notice about lines *l* and *m*? ____

Check with two of your neighbors to see if they have a similar result. (Recall that parallel lines have the same slope.) Now compare the structure of these two equations:

line *l*: 40*D* + 48 *E* = 840 line *m*: 40 *D* + 48 *E* = *your* lowest total daily wages

Angelina used the point (12,9) to generate total daily wages of $912.

21 What equation did she use to determine these total daily wages? ______________________________________

22 Where do you think Angelina's line is with respect to line *l*? ____

23 Where is her line with respect to *your* line *m*? ____

Michael used the point (5,5) to compute total daily wages of $440. Add the graph of Michael's line to your graph

24 Is Michael's line parallel to line *l*? ________

25 Could Michael's line contain the optimal solution? ________ Why or why not? _______________________________

The Corner Principle

On your graph, try to draw the line that represents the optimal solution. Remember that the "optimal line" will intersect the feasible region, yet have the lowest possible total daily wages. Notice that any other line parallel to the optimal line which has lower total daily wages fails to intersect the feasible region, while any other line parallel to the optimal line which intersects the feasible region has higher total daily wages.

> ***Optimal Solution*:** the set of values of the decision variables which satisfies all of the constraints and achieves the goal of minimizing (or maximizing) the objective function

The *optimal solution* occurs at the point where the optimal line intersects the *feasible region*. In this case, that point has the coordinates (8,8).

26 What does this point mean in terms of the problem? ____

27 What is the lowest total daily wages? ____

> If there is a unique solution to a linear programming problem, it must occur at one of the corner points of the feasible region.

28 How would you describe the location of the point (8,8) in the feasible region? ____

29 Thinking about the original problem situation, why does this solution make sense? ______________________________________

The point (8,8) is called a *corner point* of the feasible region. The feasible region for this problem has one other corner point.

30 What are the coordinates of the other corner point? (,)

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