

Part 1: Developing an Investment Plan

Latisha is considering placing up to \$12,000 in a combination of three different one-time investments. Each investment has a fixed investment requirement and produces a guaranteed annual return. Cold Cash requires exactly \$3,700 and yields an annual return of \$294. Dynamite Dollars requires exactly \$6,900 and yields an annual return of \$518. Super Stash requires exactly \$4,300 and yields an annual return of \$285.

1. Is it possible to invest all of the \$12,000? _____
2. What are the requirements of the investment options in the above example?

3. Give an example of one possible investment plan.

4. What makes a plan feasible (possible) or infeasible (impossible)?

5. Calculate the annual return of the investment plan in step 3.

6. Why did you choose this specific investment plan?



Share your plan with two other classmates and discuss the differences and similarities of your choices.

7. As a group, write down all feasible combinations of investments.

8. How did you organize your work in a way that ensured consideration of every possible investment plan?

9. Which plan produces the maximum annual return? _____
10. Assume there was one more investment option added to the three above. How would this affect the problem-solving process?

Part 2: Latisha Develops an Investment Plan

Uncle George and Aunt Lucille have given their niece Latisha \$15,000 toward her incidental college expenses. She has several choices as to how to manage the money: a) Hold the cash under her mattress. b) Put the money in a savings account at 2.3% APR (annual percentage rate) or c) Invest the money in funds that require a fixed one-time investment and a guaranteed annual return.

- How much is the annual interest on \$15,000 if 2.3% is the APR? _____

Latisha has decided to explore investing the \$15,000 and using the annual income towards her college expenses. She has chosen four different funds as possible investments, as shown in the table below. Each fund requires investing an exact amount and produces a guaranteed annual return. She decides to spend any funds not invested on new clothes for college. She will consider all of the possibilities for allocating her funds, and calculate the corresponding returns.

- Make a conjecture as to which combination of investments will maximize her annual return.



A spreadsheet is a tool that can be used to evaluate each of the potential investment plans. Enter the following headings and data in a spreadsheet:

	A	B	C	D	E	F
1	Investment	Amount Invested	Annual Income	0-1 decision variable	Actual Investment	Actual Income
2	Fabulous Funds	7900	588			
3	Cold Cash	3700	294			
4	Dynamite Dollars	6900	518			
5	Super Stash	4300	285			
6				Is this sum feasible? →		

A 0-1 decision variable is a variable that only takes on the values of 0 or 1. It is used in column D to indicate whether or not the investment is made. A zero indicates that the investment was *not* made and a one represents making that investment. Use the following set of questions to guide you in using the spreadsheet to test each investment plan.

0-1

- How do you determine if a particular combination of investments is feasible? _____

- Enter a one in the 0-1 decision variable column opposite each and every investment. Describe the investment plan that this represents. _____

- Consider the information in columns B, C, and D. Which columns would you use and what operations would you perform to calculate the results for column E? _____

Use the operations you identified to define the cell values in the Actual Investment column. Then use the sum function to determine the column total. Do the same for the last column.

6. What is the relationship of the sum of the investments in column E to \$15,000, the total amount Latisha was given? _____

7. Is this investment plan feasible? Justify your conclusion. _____

Return to the 0-1 decision variable column, and enter a zero into the first cell. Notice that the spreadsheet will automatically recalculate columns E and F. (*That’s a major advantage of using a spreadsheet.*)

8. Entering a zero in the 0-1 decision variable column changes the investment plan. Describe this new investment plan. _____

If the investment plan is feasible, record the sum given in column F and the combination of investments that yielded that return in a table you create similar to the one below. If the plan is infeasible, write “infeasible” in place of the total annual income.

Sample table

Investment Plan	Total Annual Income
1. Fab Funds, Cold Cash, Dynamite Dollars, Super Stash	Infeasible
2.	
3.	

9. How would you manipulate the spreadsheet to evaluate every investment plan that includes exactly three investments out of the four? _____

Implement your strategy from number 9 and record the results in your table. As before, if a plan is infeasible, write “infeasible” in place of the total annual income.

Repeat the analysis for all investment plans that involve making exactly two of the investments. Record each result in the table you created.

10. What combination of investments maximizes the return on her \$15,000? _____

11. Compare your initial conjecture with the recorded results of the tests. _____

12. Which of Latisha's three original choices a, b, or c provides the greatest benefit? _____

13. What other investment plans exist and why is there no need to fully evaluate them? _____

14. What is the goal of testing all the possibilities? _____

15. If Latisha had 5 possible investments, how many potential combinations of investments would there be? _____

16. If Latisha increased the number of potential investments to more than 5, explain why it would be difficult for her to make a conjecture about the best investment plan. _____

17. List the advantages of using a spreadsheet for this exercise. _____

In order to formulate and analyze larger, more complex problems of this nature, operations researchers define an *objective function* and *constraints* for this problem by utilizing *0-1 decision variables*. A 0-1 decision variable is a variable whose domain is {0,1}. These variables may be used to formulate problems involving a series of yes-no decisions. For example, suppose we let:

$$f = \begin{cases} 0, & \text{if no money is invested in Fabulous Funds, or} \\ 1, & \text{if \$7800 is invested in Fabulous Funds.} \end{cases}$$

18. What are the possible values of the expression $7800f$? _____
 What does this expression represent? _____

19. What are the possible values of the expression $588f$? _____
 What does this expression represent? _____

Now use the letters *c*, *d*, and *s* to represent 0-1 decision variables that correspond to the other three available investments and let *A* represent the total annual return from the investment plan.

20. Write an expression for *A* in terms of *f*, *c*, *d*, and *s*.
A = _____

21. Write an expression for the total money invested in terms of *f*, *c*, *d*, and *s*.

22. What limit is there on this total? _____

23. Express this constraint as an inequality.

24. Why is this formulation not necessary in order to complete the earlier spreadsheet activity?

25. What would make such a formulation necessary?

News From the World of Operations Research¹

<p>Seattle Group Uses OR Technique to Assign Season Tickets</p> <p>Large Increase in Satisfaction Reported</p>	<p>Portfolio Construction at Grantham, Mayo, Van Otterloo & Company</p> <p>Grantham, Mayo, Van Otterloo and Company of Boston recently developed a mixed-integer programming process to assist in managing a large investment portfolio of over \$8 billion in assets. This allowed the company to greatly reduce the number of different items in the portfolio, as well as the number of stock trades necessary to maintain the portfolio. As a result the company reduced annual costs to investors by at least \$4 million per year.</p>	<p>US Army Base Realignment and Closure</p> <p>BRACAS saves \$233 million</p>
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¹ Please visit www.hsor.org to view more detailed write-ups of these and other cases.

Extension: Latisha Develops an Investment Plan with a CD Option

Uncle George and Aunt Lucille did not want Latisha to use any of the funds not invested for clothing. They suggested that she place those funds in a savings account instead. Suppose the savings account pays 2.3% APR.

1. Let x represent these excess funds not yet invested. How is the decision variable x different from the decision variables f , c , d , and s .

2. Rewrite an expression for A in terms of x , f , c , d , and s .

$A =$ -----

3. Write the new \$15,000 constraint.

4. How is this constraint different from the original constraint?

5. Find the optimal solution.

Homework

1. Assigning Swimmers to the Medley Relay

Smith, Braun, Cisek, and Rivera are the four fastest swimmers on the Johnson High School swimming team. They will make up the 400-yard medley relay team. The medley consists of four different 100-yard legs in which a different competitor swims the backstroke, butterfly, breaststroke, and freestyle. Their coach, Mr. Timer, must decide who should swim each leg of the relay. The average times for each of these swimmers in each event are listed below in Table 1. (Should we replace yards by meters?)

Table 1

	free	back	breast	fly
Smith	47.59	51.67	58.31	49.18
Braun	47.31	51.62	58.95	49.89
Cisek	48.29	52.59	58.92	49.13
Rivera	48.53	53.34	59.38	50.59

Table 2

	free	back	breast	fly	
Smith	1	0	0	0	1
Braun	0	1	0	0	1
Cisek	0	0	1	0	1
Rivera	0	0	0	1	1
	1	1	1	1	

- In how many ways can Coach Timer assign the four swimmers to the four legs of the relay?

- Coach Timer has constructed Table 2 to help him decide which swimmer should swim each leg of the relay. What do you think the entries next to each swimmer’s name in Table 2 mean?

- What do you think is the meaning of the row of 1s at the bottom and the column of 1s at the right of Table 2? _____
- In order to decide how to assign the swimmers to the legs of the relay, Coach Timer must assign some sort of numerical score to each possibility. If Smith swims the freestyle leg, Braun the backstroke, Cisek the breaststroke, and Rivera the butterfly, what number would make sense to use as the score for that particular assignment?

Table 3

	free	back	breast	fly
Smith	47.59	0	0	0
Braun	0	51.62	0	0
Cisek	0	0	58.92	0
Rivera	0	0	0	50.59

208.7

- Coach Timer developed Table 3 to compute the numerical score for each particular assignment of swimmers to legs of the relay. Why is there only one non-zero entry in each row and in each column of Table 3? _____

- What do the non-zero entries mean? _____
- What is the meaning of the number 208.7, which appears below the table? _____

8. How could Table 3 be constructed from Tables 1 and 2? _____

Copy the information in Tables 1 and 2 into a spreadsheet and use that information to construct a table similar to Table 3. Then, manipulate the spreadsheet you have created to investigate all of the possible assignments of swimmers to legs of the relay.

9. Which assignment is optimal? _____

10. How would this assignment problem be complicated if the coach had five swimmers available to assign to the four legs of the relay? _____

2. Opening School Facilities

A local school district has four high schools. It runs an evening program in which it opens the gymnasium and swimming pool for use by residents of the district. The school board has determined that it can only afford to open 3 of the 4 schools for this program. The school board would also like to ensure that no resident would have to travel more than 2.5 miles to reach the nearest open facility. The school district is divided into 12 geographic regions, and Table 4 below shows the maximum distance in miles from each of the geographic regions to each of the four high schools.

Table 4: Distances from Regions to Schools

	REGION											
	1	2	3	4	5	6	7	8	9	10	11	12
Adams H.S.	1.2	1.3	2.3	3.3	1.5	1.6	2.6	3.6	2.5	2.6	3.6	4.6
Jefferson H.S.	3.1	2.1	1.1	2.2	3.3	2.3	1.2	2.2	4.3	3.3	2.2	3.2
Madison H.S.	4.5	3.5	2.3	3.4	3.5	2.5	1.3	2.4	3.3	2.3	1.2	2.4
Washington H.S.	2.4	2.5	3.5	4.5	1.4	1.5	2.5	3.5	1.5	1.4	2.4	3.4

The Director of School Facilities created the matrix shown as Table 5 in order to learn whether the school board can meet both of the stated criteria: 1) open only 3 of the 4 schools and 2) the greatest distance to any open school would be 2.5 miles or less.

Table 5: 2.5 Mile Criterion

	REGION											
	1	2	3	4	5	6	7	8	9	10	11	12
Adams H.S.	1	1	1	0	1	1	0	0	1	0	0	0
Jefferson H.S.	0	1	1	1	0	1	1	1	0	0	1	0
Madison H.S.	0	0	1	0	0	1	1	1	0	1	1	1
Washington H.S.	1	1	0	0	1	1	1	0	1	1	1	0

1. What is the significance of the 0s and 1s in each row of the matrix? _____

2. Which school(s) cover the 2.5 mile criterion for the residents of Region 1?

3. Are there any regions for which the 2.5 mile criterion is not covered by any of the schools?

4. Are there any regions for which the 2.5 mile criterion is covered by only one school?

5. Does your answer to question 4 tell you anything about which schools must be open?

6. Is it possible to cover the 2.5 mile criterion for every region if only 3 schools are open? _____ If so, which three schools should be open? _____

The city would like to determine how much the standard for coverage would have to be increased above 2.5 miles before the city would need only two schools to cover all 12 regions. Do this analysis by increasing the coverage standard in 0.5 mile increments and recreating a table similar to Table 5.

7. What would the distance criterion have to be, in order for two schools to cover all 12 regions? _____
8. Which two schools would have to be opened to cover the distance criterion you found in question 7?
