Fuel Blending at Jurassic Oil:
How to Minimize the Cost of a Quality Blend

Teacher Resources

Extensions

The four exercises on the following pages extend the base problem. These might be assigned as homework examples at the teacher's discretion. In order to develop a deeper understanding of real-life applications of mathematics, students should be allowed to discuss these extensions, as well as other sorts of situations and the differing constraints they would entail.

The first three extensions are relatively straightforward. The fourth extension discusses the nonlinearity of octane. A graphing calculator is necessary to complete the fourth extension and a bit more structure has been provided to assist students.

Vapor Pressure

Extension 1: Leave all the conditions the same as in the sample problem except for the vapor pressure. Rewrite that problem as "The first grade has an octane number of 92, a vapor pressure of 5.5 psi and contains 0.4% sulfur and the second grade has an octane number of 85, a vapor pressure of 4.5 psi and contains 0.25% sulfur." Leave all of the other characteristics of the problem the same. Solve this new problem.

Changing a Requirement of the Blend

Extension 2: Leave all the conditions the same as in the sample problem except the third characteristic of the blend. Rewrite this characteristic as "contains less than 0.35% sulfur." Solve the problem using this new set of constraints.

A Different Blend

Extension 3: Leave all the conditions the same as in the sample problem except the first and third characteristics of the blend. Reword them as "an octane number higher than 89" and "contains less than 0.35% sulfur." Solve the problem using these constraints.

An Extension for the Graphing Calculator

Extension 4: Over the years, octane blending studies have shown that the octane number, y_1, of the blend of gasolines in the Jurassic Oils problem is:

$$y_1 = \frac{11{,}040 - 7x}{120} - \frac{2.8x(120 - x)}{120^2},$$

where x = the number (in 1000s) of bbl of Allif gasoline and 120 - x = the number (in 1000s) of bbl of HyOctane gasoline.

1. What should the graph of this function look like? Why?

2. Using a graphing calculator, graph this function with [0,120] x [85,92] as the viewing window.

3. Why does this viewing window make sense for the Jurassic Oils problem?

Did you accurately predict what the graph of this function looks like?

If the behavior of octane numbers were linear, then the function would be $y_2 = \frac{11{,}040 - 7x}{120}$.

4. In the same viewing window, graph y_1 and y_2.

5. Where do these two graphs intersect?

6. Now graph both of these functions using [30,90] x [85,92] for the viewing window.

7. Why does $30 \leq x \leq 90$ make sense for this problem?

8. For x = 30, 40, 50, 60, 70, 80, and 90, find the difference between y_1 and y_2. Where does it appear that this difference is greatest? Why does that make sense?

9. In the same viewing window, graph y_1 and y_2 along with y_3 = 89, the minimum octane number required of the blend. Then find the point of intersection of the two straight lines.

10. What is the significance of the x-value of this point?

11. Using this x-value, find the value of y_1. What does the value of y_1 represent?

Since x = 51,428.6 bbl of Allif gasoline in the blend actually produces an octane number less than 89, it cannot be the optimal solution to the Jurassic Oils problem.

12. Where in the system of graphs of y_1, y_2 and y_3 is the optimal solution?

13. What is the optimal solution?

HOMEWORK PROBLEMS

1. A fruit grower can use two types of fertilizer in his orange grove, Brand A and Brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chlorine in a bag of each brand are given in the table. Tests indicate that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chlorine.

Pounds per Bag	Brand A	Brand B
Nitrogen	8	3
Phosphoric Acid	4	4
Chlorine	2	1

   If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?

2. A dietitian in a hospital is to arrange a special diet composed of two foods, M and N. Each ounce of food M contains 30 units of calcium, 10 units of iron, 10 units of vitamin A, and 8 units of cholesterol. Each ounce of food N contains 10 units of calcium, 10 units of iron, 30 units of vitamin A, and 4 units of cholesterol. If the minimum daily requirements are 360 units of calcium, 160 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to meet the minimum requirements, and, at the same time, minimize the cholesterol intake? What is the minimum cholesterol intake?

3. Each week, the DeeLite Milk Company gets milk from two dairies and then blends the milk to get the desired amount of butterfat for the company's premier product. Dairy A can supply at most 700 gallons of milk averaging 3.7% butterfat and costing $1.50 per gal. Dairy B can supply milk averaging 3.2% butterfat costing $1.25 per gal. How much milk from each supplier should DeeLite use to get 1000 gallons of milk with at least 3.5% butterfat?

Solutions to the Extensions

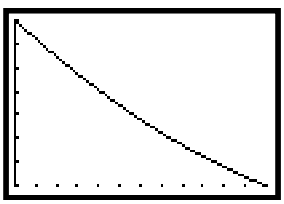
Extension 1: No solution is possible. The "new" vapor pressure constraint: H - A ≤ 0 . The feasible region cannot be drawn to meet **all** of the constraints. When the feasible region is an empty set, there does not exist an optimal solution to the proposed situation. You may wish to have the students sketch the graphs and discuss what could be done in this case.

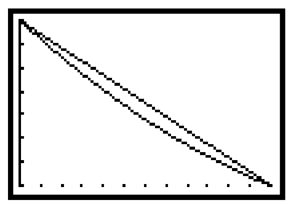
Extension 2: 68,571.4 barrels from HyOctane and 51,428.6 barrels from Allif Oil. This is the same solution as the base problem in the module. The feasible region is a line segment again, but, in this case, the second endpoint is **not** part of the feasible region. (The "new" sulfur constraint is H - 2A < 0 .) The second endpoint was not the optimal solution to the base problem, so eliminating it from the feasible region does not change the optimal solution.

Extension 3: (The "new" octane constraint is 3H - 4A > 0 and the "new" sulfur constraint is H - 2A < 0.) In this problem, the feasible region is a line segment whose endpoints are not part of the feasible region. In these situations, no discrete optimal solution exists.

Extension 4:

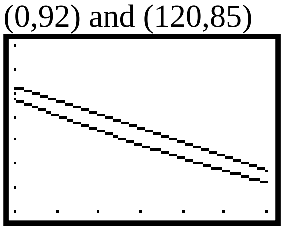
1. An inverted parabola; the function is second degree in *x*.

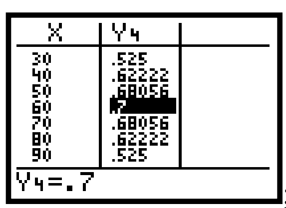
2. 

4. 

3. The total number of bbl of gasoline to be blended is 120,000; the octane numbers of the two grades to be blended are 85 and 92.

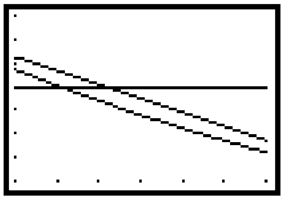
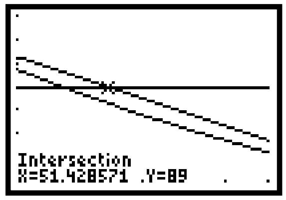
5. (0,92) and (120,85)

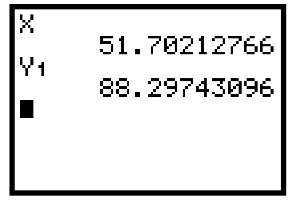
6. 

8. 

; $x = 60$; if $y_1 = y_2$ at the extremes where none of one of the grades is used in the blend, it makes sense that the greatest difference would occur when the blend uses 50% of each grade.

7. For each grade of gasoline, at least 30,000, but not more than 90,000 bbl can be used in the blend.

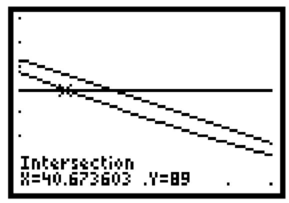
9.  

11. 

10. This value of *x* represents the optimal solution found earlier for the sample problem.

The true octane number would be about 88.3

12. At the intersection of the horizontal line (y_3) and the parabola (y_1).

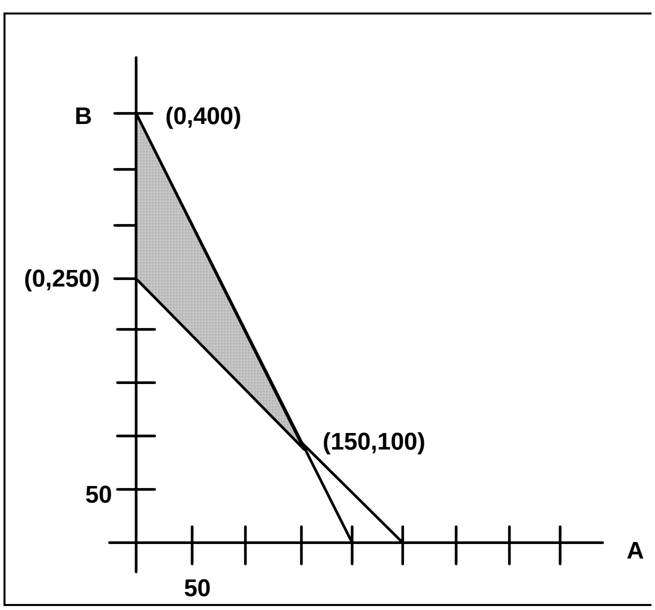
13. 

n.b., if *x* = 40,673.6 bbl are purchased from Allif, then 120 - *x* = 79,326.4 bbl must be purchased from HyOctane.

Solutions to Homework Problems:

1 If N represents the number of lbs of nitrogen, and A and B represent the number of lbs of Brand A and Brand B fertilizer, respectively, then the problem can be formulated as:

maximize the objective function, N = 8A + 3B
subject to the constraints 4A + 4B $\geq$ 1000 and 2A + B $\leq$ 400.



N = 8A + 3B
N = 8(0) + 3(400) = 0 + 1200 = 1200

N = 8A + 3B
N = 8(0) + 3(250) = 0 + 750 = 750

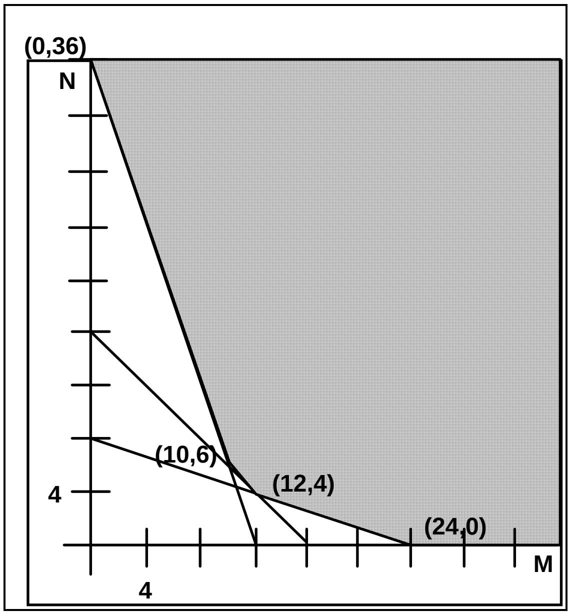
N = 8A + 3B
N = 8(150)+3(100) = 1200 + 300 = 1500

So the optimal solution is (150,100); i.e., 150 lbs of Brand A and 100 lbs of Brand B

2 Let C = the number of units of cholesterol and let M and N represent the numbers of oz of food M and food N, respectively. Then the problem can be formulated as:

minimize the objective function, $C = 8M + 4N$, subject to the constraints

$30M + 10N \geq 360$ ($3M + N \geq 36$),
$10M + 10N \geq 160$ ($M + N \geq 16$), and
$10M + 30N \geq 240$ ($M + 3N \geq 24$).



$C = 8M + 4N$
$C = 8(0) + 4(36) = 0 + 144 = 144$

$C = 8M + 4N$
$C = 8(10) + 4(6) = 80 + 24 = 104$

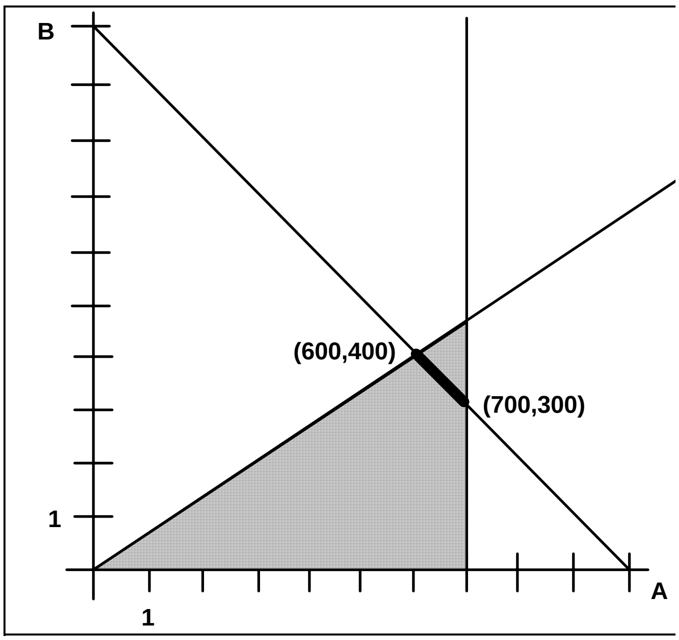
$C = 8M + 4N$
$C = 8(12) + 4(4) = 96 + 16 = 112$

$C = 8M + 4N$
$C = 8(24) + 4(0) = 192 + 0 = 192$

So the optimal solution is (10,6); i.e., 10 oz of food M and 6 oz of food N.

3 Let C = the cost of the milk blend and let A and B represent the number of gal of milk purchased from Dairy A and Dairy B, respectively. Then:

minimize C = 1.5 A + 1.25 B subject to the constraints:
$A \leq 700$
$A + B = 1000$
$\frac{3.7A + 3.2B}{A + B} \geq 3.5 \; (B \leq \frac{2}{3} A)$



Notice that the feasible "region" is the line segment whose endpoints are (600,400) and (700,300).

C = 1.5 A + 1.25 B
C = 1.5 (700) + 1.25 (300)
C = 1050 + 375 = 1425

C = 1.5 A + 1.25 B
C = 1.5 (600) + 1.25 (400)
C = 900 + 500 = 1400

So, the optimal solution is (600,400); i.e., purchase 600 gal from Dairy A and 400 gal from Dairy B. The minimum cost is then $1400.