Fuel Blending at Jurassic Oil
How to Minimize the Cost of a Quality Blend

Mr. Pete Troleum is the manager of an oil refinery for the Jurassic Oil Corporation. At the refinery, Mr. Troleum has available two grades of gasoline which must be blended at minimum cost before they are sold. The first grade, from HyOctane, Inc., has an octane number of 92, a vapor pressure of 4.5 psi at 100°F, and contains 0.4 % sulfur. The second grade, from Allif Oil, has an octane number of 85, a vapor pressure of 5.5 psi at 100°F, and contains 0.25 % sulfur. Jurassic Oil wants to produce a blend with the following characteristics:

1. an octane number of at least 89
2. a vapor pressure of no more than 5 psi at 100°F, and
3. a sulfur content of no more than 0.35 %.

The company wants to produce exactly 120,000 barrels of the blend each week. They can purchase up to 90,000 barrels of each grade per week. Mr. Troleum is able to purchase the 92 octane grade from HyOctane, Inc., at a cost of $20 per barrel, and the 85 octane grade from Allif Oil, at a cost of $15 per barrel. He must decide how many barrels of each grade should be used in the blend in order to minimize the production cost. Answering the following will help Mr. Troleum with his problem:

1 What decisions must Pete make? ____

In this activity, we will determine the exact quantity of each grade of gasoline that Mr. Troleum needs to meet the conditions of the blend, while also keeping in mind the desire to minimize the cost of purchasing the two grades of gasoline used to make the blend.

Decision Variables:

Before we can compute the solution to Mr. Troleum's problem, we must first consider the decisions that must be made. Mr. Troleum needs to determine the answers to the following questions: 1) How many barrels of the 85 octane gasoline should be used in the blend?, and 2) How many barrels of the 92 octane gasoline should be used in the blend? We will use ***decision variables*** to represent these quantities. All linear programming problems have decision variables. Our problem contains two decision variables. They can be represented by:

Decision variables: a set of variable quantities completely describing the decisions to be made.

A= the number of bbl of 85 octane gasoline needed from Allif Oil for the blend Pete wants.
H= the number of bbl of 92 octane gasoline needed from HyOctane, Inc. for the blend Pete wants.

2 If Pete purchases 90,000 bbl of gasoline from Allif Oil how much will it cost? __________________________

3 If Pete purchases 30,000 bbl of gasoline from HyOctane, Inc. how much will it cost? __________________________

4 What would be the total cost of these 120,000 bbl? __________________________

5 Write an expression to represent the *total gasoline* purchase cost for Pete's blend if he buys **A** barrels of gasoline from Allif Oil and **H** barrels of gasoline from HyOctane, Inc.

____

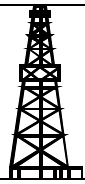
Objective Function:

In any linear programming problem the decision maker wants to optimize some function of the decision variables. For example, the decision maker may want to minimize costs or maximize profit. The function which expresses this objective mathematically is called the ***objective function.*** In Pete's gasoline blending problem, he wants to minimize the cost of buying the gasoline needed to produce the blend. Therefore, the function you wrote for question 5 is the objective function for this problem.

Objective Function: a quantity to be maximized or minimized which is defined in terms of the decision variables.

Constraints:

Constraints: restrictions on the values of one or more of the decision variables.



Any restriction on one or more of the decision variables is called a ***constraint.*** Our constraints come from information in the sample problem. In all, there are six constraints. Two of them deal with the maximum amount of gasoline that Pete can purchase from the two companies each week.

6 Write two inequality statements representing the gasoline purchase restrictions.

_________________________ and _________________________

7 Write the constraint that restricts the *number of barrels* of the blend that Jurassic Oil will produce each week.

____

8 What makes your answer in 7 different from the other constraints? ______________

9 Look back at your answer to 4. What is the *average* cost per barrel of those 120,000 bbl?

average cost per barrel = **$**___________________

10 Write a sentence or two explaining why the average cost per bbl that you computed in question 9 is *not* $17.50.

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In question 9**,** we found the average cost per barrel. The same answer could also be found using a formula involving weighted means, a concept used frequently in the field of statistics. The weighted means formula is appropriate whenever the components of a mean do not contribute equally to it. The formula would appear as follows**:**

$$\textbf{\textit{average} cost per bbl} = \frac{\textbf{(A) (cost per bbl of A) + (H) (cost per bbl of H)}}{\textbf{(A + H)}}$$

[**Note: A** and **H** are the number of barrels of each grade of gasoline.]

Try question 9 again, this time using the formula, with **A** = 90,000 and **H** = 30,000. Did you find the same average cost per barrel?

11 Suppose Pete wants to compute the average cost per barrel for an undetermined quantity of each octane grade (i.e., he does not have a value for **A** and **H**). Write an expression for the average cost per barrel of the blend.

average cost per barrel = ______________________________________ .

Now we will use the weighted means formula to express the other three constraints.

12 The vapor pressure of the blended gasoline must be no more than _______psi . This is a combination of a vapor pressure of

_______psi from the gasoline purchased from Allif Oil Company and _______psi from the gasoline purchased from HyOctane, Inc.

Using the weighted means formula, this constraint can be written as the following inequality:

$$\frac{\mathbf{A}(\_\_\_\_\_) + \mathbf{H}(\_\_\_\_\_)}{\mathbf{A + H}} \leq \_\_\_\_\_\_$$

13 Use the same approach for determining the inequalities for sulfur content and octane rating.

<u>sulfur content inequality</u> **<u>octane level inequality</u>**

14 For ease in graphing, rewrite these inequalities in a standard linear form:

vapor pressure inequality	sulfur content inequality	octane level inequality

Optimal Solution:

The ***optimal solution*** is just the best solution to a particular problem. In this case, the values of **A** and **H** which satisfy all of the constraints and minimize the weekly total gasoline purchase cost is the *optimal solution*. The objective function, **C** = 15 **A** + 20 **H** , defines the weekly cost, where **A** represents the number of barrels purchased from Allif Oil each week, and **H** represents the number of barrels purchased from HyOctane, Inc. each week. So, how *do* we find the lowest weekly cost that satisfies all of the constraints?

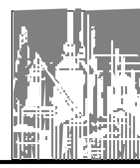
First, we must find *all* possible weekly costs that satisfy all of the constraints; but how many possibilities are there? If we represent pairs of values of **A** and **H** as points in a coordinate system, the set of all such points that satisfy all of the constraints is called the ***feasible region***. Graphically, the feasible region is the region of the coordinate system bounded by the graphs of the constraints. The feasible region can be empty, as small as a single point, or contain an infinite number of points.

The feasible region for Pete's blending problem is the set of points bounded by the following constraints:

$H \leq 90{,}000$ $\quad$ $A \leq 90{,}000$ $\quad$ $A + H = 120{,}000$ $\quad$ $4A - 3H \leq 0$ $\quad$ $A - H \leq 0$ $\quad$ $2A - H \geq 0$

We will find the feasible region for Pete's blending problem by graphing the six constraints on the same coordinate axes. Although it does not matter how you label the axes, for ease in comparison, let the horizontal axis represent the number of barrels of gasoline purchased from Allif Oil (**A**), and let the vertical axis represent the number of barrels of gasoline purchased from HyOctane, Inc. (**H**). On a sheet of graph paper, graph each of the six constraints. You may find it helpful to express all of the constraints in slope intercept form (i.e., **H**= m**A** + b).

On your graph, indicate the region of the coordinate system containing all of the points satisfying all of the constraints. The feasible region is the set of all points which *could* be the optimal solution, because they satisfy all of the constraints.



Feasible region: the set of all points which satisfy all of the constraints.

15 How could you describe the feasible region ? ____

16 Which constraints are used to determine the feasible region? ____

Sometimes a constraint adds nothing to the feasible region. Constraints like this could be removed from the system, and the feasible region would remain unchanged.

17 Are any of the constraints in Pete's blending problem unnecessary in determining the feasible region? If so, which?

____

Constraints which add nothing to the feasible region are called ***redundant***. Redundant constraints will never form part of the boundary of the feasible region.

18 Is (50 000, 70 000) a point in the feasible region? _______ Why or why not?____

19 Find the endpoints (to the nearest tenth of a bbl) of the line segment that makes up the feasible region by either tracing on your calculator or by solving for the intersection of the equations algebraically. The endpoints are:

(__________________ , __________________) and(__________________ , __________________) .

Therefore, the feasible region contains all of the points on that segment including the two endpoints listed above. The ***optimal solution*****,** must be one of the points in the feasible region, and it is the point that minimizes the objective function (**C = 15A + 20 H),** but still meets all of the constraints.

20 Since (50 000, 70 000) is a point in the feasible region, use the objective function to find the cost for this point.

C =___________________________

21 Find another point that is in the feasible region. Use the objective function to find the cost for this second point.

Point (_________________ , _________________) **C** =__________________________

22 Now test the two endpoints that you found in question **19**:

Endpoint (_________________ , _________________) **C** =__________________________

Endpoint (_________________ , _________________) **C** =__________________________

23 Which endpoint yields the lower cost? (_________________ , _________________).

To help find the optimal solution, we will use the graph of 15**A** + 20**H** = 2,150,000 (the cost you found in number 20). Add the graph of this equation to your graph of the feasible region, and label it line *l*. Now, using your answer from 23**,** graph 15**A + **20**H** = *the lower cost* and label it line *m*.

24 What do you notice about lines *l* and *m*?____.

The ***optimal line*** intersects the feasible region, yet has the lowest possible cost for the gasoline. Notice that any other line parallel to the optimal line which has a lower cost fails to intersect the feasible region. Furthermore, any other line parallel to the optimal line which intersects the feasible region has a higher cost.

25 The optimal solution occurs at the point where the optimal line intersects the feasible region. That point has coordinates

(_________________ , _________________) .

How would you describe the location of the point in the feasible region? ____

26 What does this point mean in terms of the problem?____

27 What is the lowest cost for Jurassic Oil? ____

28 What are the octane number, vapor pressure, and sulfur content of the blend? _____________________________________

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News From the World of Operations Research

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