Extensions

Four extensions are provided in the packet. The first extension is a more difficult version of the sample problem, because the network contains more nodes and arcs. The second requires the construction of a table of distances (similar to tables found on maps or in atlases) from the graph in the Extension 1. The third extension asks students to construct a weighted network from a table of distances. Finally, the fourth further complicates the Extension 1 by adding one-way streets to that network.

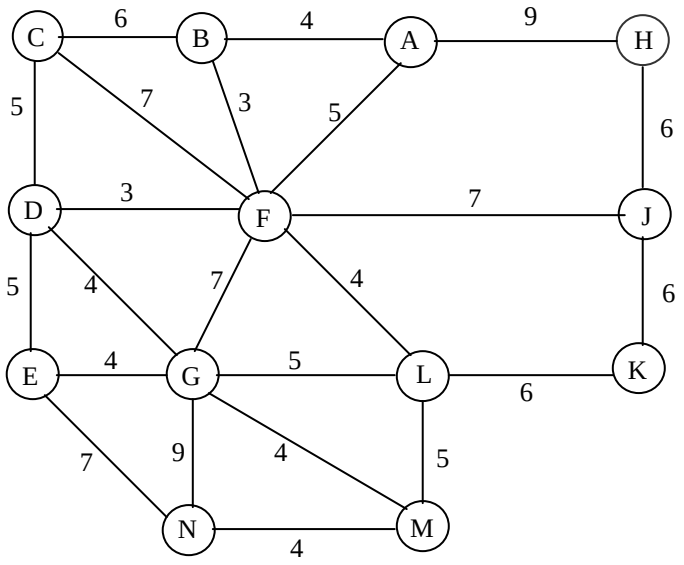
Extension 4 contains one-way streets. This aspect of the problem is captured through the concept of adjacent nodes. For example, if you are at node J in this network, node K is not an adjacent node due to the one-way arc from node K to J.

In extension 4, students can find the required shortest route without finding the shortest route to each of the other nodes. With a little extra effort, the algorithm can be continued to generate the shortest path to every other node in the network.

Extension 1: A more complicated network

Because of the success of its quick delivery time, the *Speedy Delivery* Company has expanded operations. Headquarters is still located at node A, but some additional locations have been added to the delivery area.

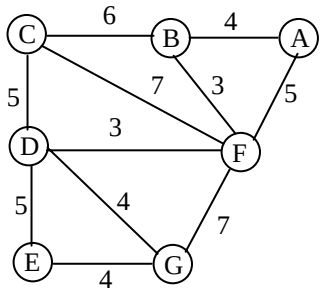
It was 3:30 P.M. and Mr. I.M. Lait has just made a delivery to a new customer at point H. Harry Upp contacts I.M. Lait and informs him that the Pane N. Butt Co. at point N, also a new customer, needs something picked up before 4 P.M. Refer to network below, and find the shortest time to go from point H to point N. Fill in the graph and the table provided according to the algorithm. Can Mr. Lait reach the company in time?



Circled Node	Adjacent Nodes (uncircled)	Path (from H)	Total Time

Extension 2: The Other Way Around

Perhaps you have seen a table showing the travel times between major cites on a map. Using the weighted network from figure 2 in the Speedy Delivery problem, make such a table. Be sure to show the travel times between every pair of nodes in the network.



	A	**B**	**C**	**D**	**E**	**F**	**G**
A	---				13		
B		---					
C			---				
D				---			
E	13				---		
F						---	
G							---

Extension 3: Cities in Michigan

Helen Wheels drives a truck "over the road" between the cities listed in the mileage table below. She would like to develop a schedule where no city-to-city mileage exceeds 100 miles.

a. Based upon the mileages given in the table, draw a graph with vertices representing cities and edges representing trips less than 100 miles. If your original graph has edges intersecting, reposition the vertices and redraw the graph to eliminate the intersections.

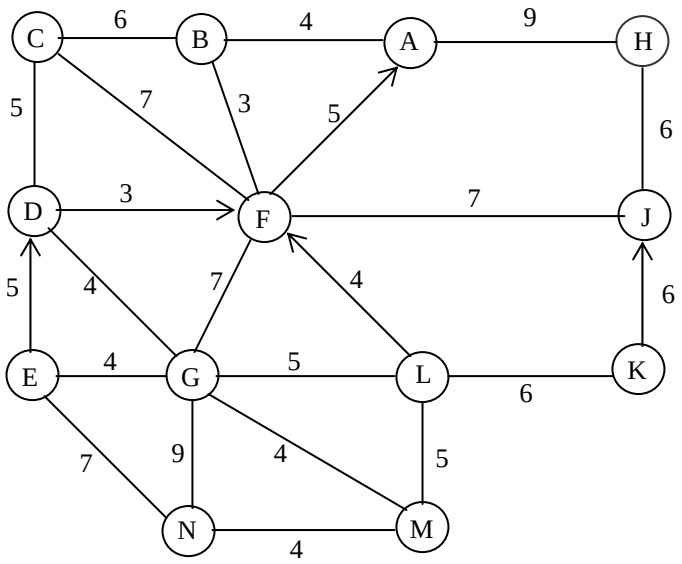
b. Using your graph find the shortest route from Kalamazoo to Flint.

c. Using your graph find the shortest route from Detroit to each city listed in the table.

	Ann Arbor	Detroit	Flint	Grand Rapids	Kalamazoo	Lansing
Ann Arbor	*	51	56	146	101	76
Detroit	51	*	62	156	136	90
Flint	56	62	*	121	134	56
Grand Rapids	146	156	121	*	50	91
Kalamazoo	101	136	134	50	*	78
Lansing	76	90	56	91	78	*

Extension 4: One-way Streets

In the network below, some of the routes are one-way streets. They are marked with arrows. Headquarters is still located at point A. It was 3:30 p.m. and Mr. I.M. Lait has just made a delivery to a new customer at point H. Harry Upp contacts I.M. Lait and informs him that the Pane N. Butt Co. at point N, also a new customer, needs something picked up before 4 p.m. Using the graph and chart below, find the shortest time to go from point H to point N. Fill in the graph and the table provided according to the algorithm. Can Mr. Lait reach the company in time?



Circled Node	Adjacent Nodes (uncircled)	Path (from H)	Total Time

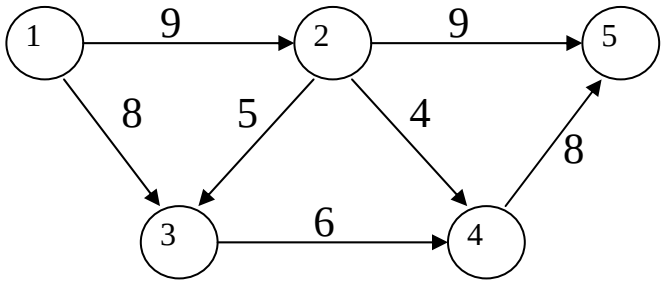
Homework Exercises

The exercises which follow are standard shortest path problems which are set in contexts taken from, or similar to, those in the case studies. You may want to provide students with some of the information in the case studies for background.

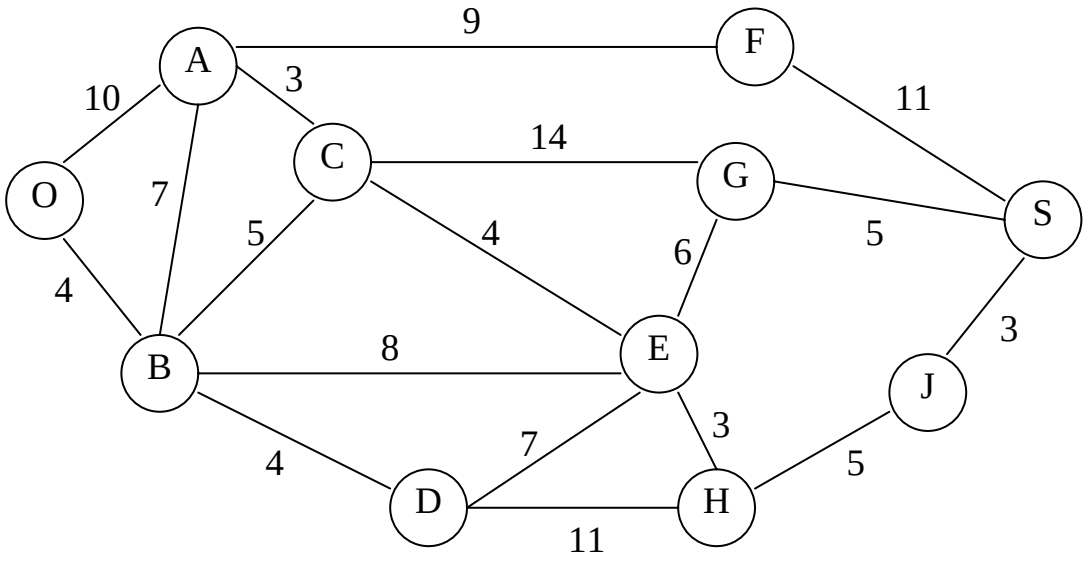
Homework problem number 1 includes the concept of one-way streets and raises the possibility that there is no route between two nodes.

Homework problem number 2 introduces a real-world context for a large network. It also asks the students to discuss how contextual issues such as time of day might affect the choice of the best travel route between a plant and a warehouse.

1. In the diagram below the arrows indicate one-way streets. "Hot Meals & Wheels" must make deliveries at locations 1 and 4. Find the shortest route from node 1 to node 4. What can be observed about the best path from node 4 to node 1?



2. The Acme Bottling Company recently had a grand opening for their new warehouse in Soda City. Nate Carbo, the bottling manager, feels that the present route being used by the trucks supplying the new warehouse can be improved. Trucks currently take the route OAFS where O is the plant at O.K.Cola Junction and S is the new warehouse. Although the roads are relatively straight and well maintained, Nate objects to the distance traveled. Recent studies indicate that the cost of transportation is greatly affected by the distances the trucks travel. Transportation costs primarily consist of maintenance, replacement, and driver wages. Because the fleet makes many trips to Soda City annually, considerable cost reductions can be expected for each mile the route is shortened. The diagram below is a representation of the road network between O.K. Cola Junction and Soda City. Although the arcs of the network are all straight lines, the actual roads may have many curves or hills. It is safe to assume that the rate of speed along any road is the same. The diagram shows the distance, in miles, of each road segment.

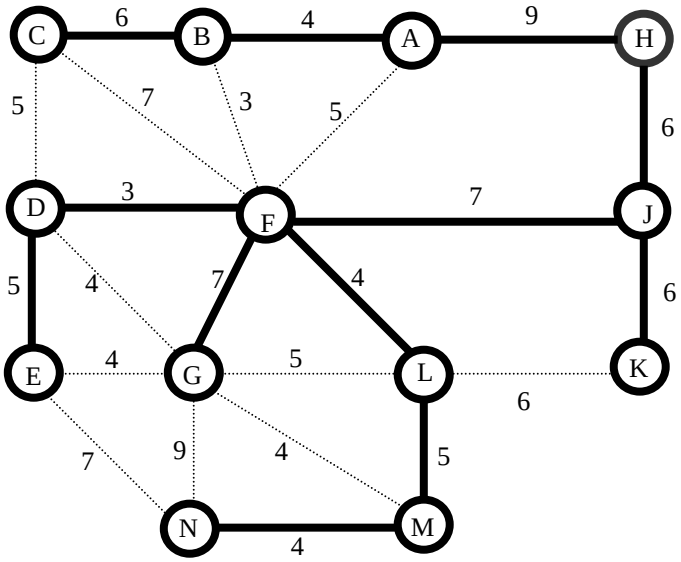


a. Use the shortest-route method to find the best route from O.K. Cola Junction to Soda City.

b. What other considerations should Mr. Carbo make before authorizing a new route? How might time of day affect the choice of preferred route?

Solution to Extensions

Extension #1



Circled Node	Adjacent Nodes (uncircled)	Path (fromH)	Total Time
H	A	HA	9
	J	HJ	6
J	F	HJ	13
	K	HJK	12
A	B	HAB	13
	~~F~~	~~HAF~~	~~14~~
K	~~L~~	~~HJKL~~	~~18~~
F	~~B~~	~~HJFB~~	~~16~~
	~~C~~	~~HJFC~~	~~20~~
	D	HJFD	16
	G	HJFG	20
	L	HJFL	17
B	C	HABC	19

D	~~C~~	~~HJFDC~~	~~21~~
	(E)	HJFDE	21
	~~G~~	~~HJFDG~~	~~20~~
L	~~G~~	~~HJFLG~~	~~22~~
	(M)	HJFLM	22
C	None		
G	~~E~~	~~HJFGE~~	~~24~~
	~~M~~	~~HJFGM~~	~~24~~
	~~N~~	~~HJFGN~~	~~29~~
E	~~N~~	~~HJFDEN~~	~~28~~
M	(N)	HJFLMN	26
N	endpoint of optimal route		

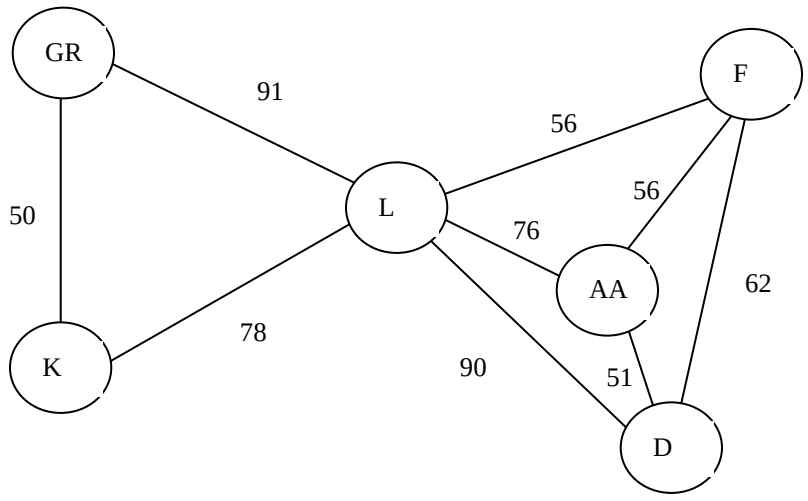
The shortest time from H to N is 26 and the path is HJFLMN.

Extension #2

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	---	4	10	8	13	5	12
B	4	---	6	6	11	3	10
C	10	6	---	5	10	7	9
D	8	6	5	---	5	3	4
E	13	11	10	5	---	8	4
F	5	3	7	3	8	---	7
G	12	10	9	4	4	7	---

Extension #3

a.)



	circled node	adjacent nodes (uncircled)	path (from A)	total distance
b.)	K	(G)	KG	50
		(L)	KL	78
	G	~~L~~	~~KGL~~	~~141~~
	L	A	KLA	154
		(F)	KLF	134
		D	KLD	168

The shortest route from Kalamazoo to Flint is 134 miles.

	circled node	adjacent nodes (uncircled)	path (from A)	total distance
c.)	D	(F)	DF	62
		(A)	DA	51
		(L)	DL	90
	A	~~F~~	~~DAF~~	~~107~~
		~~L~~	~~DAL~~	~~127~~
	F	~~L~~	~~DAFL~~	~~163~~
	L	(G)	DLG	181
		(K)	DLK	168
	K	~~G~~	~~DLKG~~	~~218~~

Detroit to Ann Arbor:	51 miles
Detroit to Flint:	62 miles
Detroit to Lansing:	90 miles
Detroit to Grand Rapids:	181 miles
Detroit to Kalamazoo:	168 miles

Extension #4

Circled Node	Adjacent Nodes (uncircled)	Path (from H)	Total Time
H	(A)	HA	9
	(J)	HJ	6
J	(F)	HJF	13
	K	wrong way on one-way street	
A	(B)	HAB	13
	F	wrong way on one-way street	
F	~~B~~	~~HJFB~~	~~16~~
	~~C~~	~~HJFC~~	~~20~~
	D	wrong way on one-way street	
	(G)	HJFG	20
	L	wrong way on one-way street	
B	(C)	HABC	19
C	(D)	HABCD	24
G	~~D~~	~~HJFGD~~	~~24~~
	(E)	HJFGE	24
	(L)	HJFGL	25
	(M)	HJFGM	24
	~~N~~	~~HJFGN~~	~~29~~
D	E	wrong way on one-way street	
E	~~N~~	~~HJFGEN~~	~~31~~
M	L	HJFGML	29
	(N)	HJFGMN	28

Answer: Path HJFGMN. Time: 28 minutes. Mr. Lait will reach the company at 3:58 pm.

Homework Solutions

1.

Circled Node	Adjacent Node	Path	Total Time
1	(2)	1 – 2	9
	(3)	1 – 3	8
3	4	1 – 3 – 4	14
2	5	1 – 2 – 5	18
	4	1 – 2 – 4	13

The shortest route from location 1 to location 4 is found by taking the path 1 – 2 – 4, which leads to a total time of 13 minutes.

2.

Circled Node	Adjacent Node	Path	Total Time
O	(A)	OA	10
	(B)	OB	4
B	~~A~~	~~OBA~~	~~11~~
	(C)	OBC	9
	(D)	OBD	8
	(E)	OBE	12
D	~~E~~	~~OBDE~~	~~15~~
	~~H~~	~~OBDH~~	~~19~~
C	~~A~~	~~OBCA~~	~~12~~
	~~E~~	~~OBCE~~	~~13~~
	~~G~~	~~OBCG~~	~~23~~
A	(F)	OAF	19
E	(G)	OBEG	18
	(H)	OBEH	15
H	(J)	OBEHJ	20
G	*S*	*OBEGS*	*23*
F	S	OAFS	30
J	*S*	*OBEHJS*	*23*

a. There are two possible paths providing the shortest route: paths OBEGS and OBEHJS. Both paths are 23 miles long. The old path (OAFS) is 30 miles long.

b. It is not possible in this network of one-way streets to travel from node 4 to node 1. Lead a discussion of experiences in which students had difficulty in getting to their destinations because the streets seemed always to be one-way in the wrong direction.