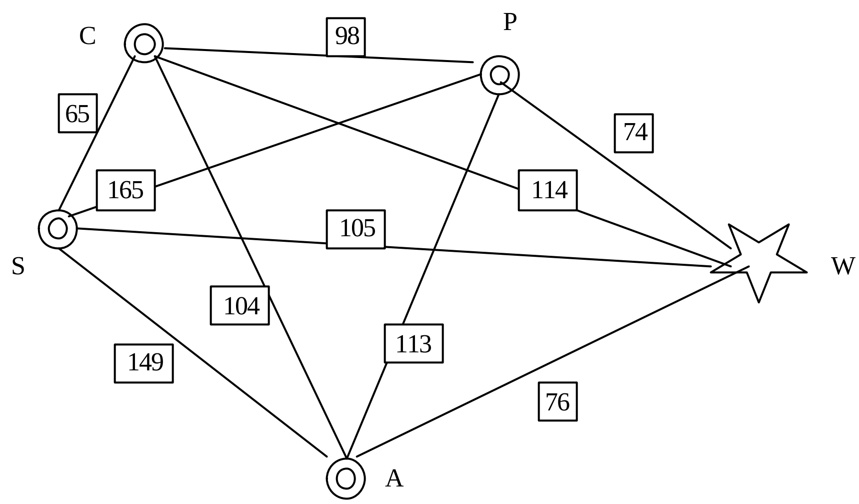
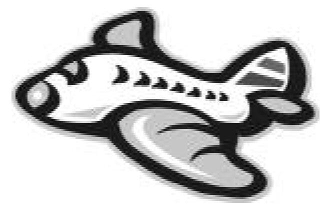
Outel Semiconductor's Recruiting Circuit: Finding the Cheapest Route

Mr. Ira Cruit works in the human resource department of Outel Semiconductor. His office is in the Washington, DC area. He has the responsibility for recruiting top notch graduating seniors from 1) Carnegie-Mellon, in Pittsburgh, PA, 2) Northwestern University in a suburb of Chicago, IL, 3) Washington University in St. Louis, MO, and 4) Georgia Tech, in Atlanta, GA. He plans to visit them in one four-day period and return home to Washington DC. Below is a map showing the cheapest available one-way fares between every possible pair of cities in either direction. Mr. Cruit wants to determine the cheapest total travel cost for his trip.





Washington
Pittsburgh
Chicago
St. Louis
Atlanta

The figure above is called a **network**. A **Hamiltonian Path** is a path that visits every stop or **node** exactly once. If this path returns to the starting point, making a closed loop, then the path is a **Hamiltonian Circuit**. Mr. Cruit's problem involves finding a route starting at W passing through every other node exactly once and returning to W, creating a Hamiltonian Circuit.

One method Mr. Cruit can use to find the cheapest itinerary is **brute force**. He can list and then calculate the cost of every possible Hamiltonian circuit. Let's create the list in a systematic fashion to be sure that we consider every possible circuit.

Circuit #	Start	1	2	3	4	Return	Circuit Sequence	Total Cost
1	W	P	C	S	A	W	WPCSAW	74+98+65+149+76 = 462
2	"	"	"	A	S	"	WPCASW	
3	"	"	S	A	C	"	WPSACW	
4	"	"	"	C	A	"	WPSCAW	
5	"	"	A	S	C	"	WPASCW	
6	"	"	"	C	S	"	WPACSW	
7	**"**	**C**	**P**	**S**	**A**	**"**	**WCPSAW**	
8	"	"				"		
9	"	"				"		
10	"	"				"		
11	"	"				"		
12	"	"				"		
13								
14								
15								
16								
17								
18								
19								
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21								
22								
23								
24								

1. Complete circuits 8 through 12 by placing node letters in columns 2, 3 and 4. Each of the circuits will start with WC and return back to W. Record the circuit sequence.
2. Repeat step 1 for circuits numbered 13 through 18. (Start each of the circuits with WS.)
   Repeat step 1 for circuits numbered 19 through 24. (Start each of the circuits with WA.)
3. Among the list of circuits numbered 8 through 12, you should have identified a circuit **WCSAPW**. If you travel this circuit in reverse order, what circuit would it be? _______ Where in the list does this circuit already appear? ______
4. A complete circuit can be traveled in either direction and the two circuits are equivalent. Find any other duplicate circuits among the list of circuits 8 through 12. Cross out each of the duplicates in the table.
5. Repeat step 4 for circuits numbered 13 through 18.
   Repeat step 4 for circuits numbered 19 through 24.

6. How many unique circuits remain? _____________
   How many circuits were originally created? __________
7. How does the number of unique circuits compare to the total number of circuits?
   ____
8. The total cost of circuit 1 is $462 and its calculation is recorded in the table. Calculate and record in the table the cost for each of the remaining circuits.
9. Which circuit would be the cheapest? ______________
   What is its cost? ______

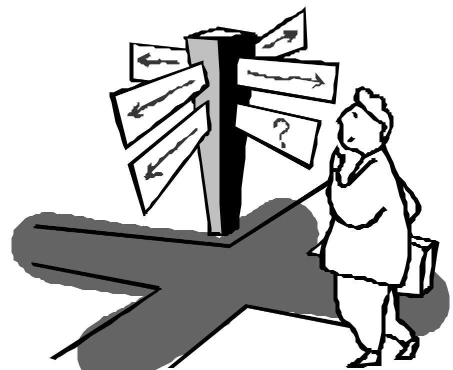
The brute force method involves identifying every unique circuit. It is important to understand the relationship between the number of nodes and the number of unique circuits. Let's begin developing a formula that represents this relationship.

10. How many nodes can you travel to from W? ______
11. After you have chosen the second node in your circuit, how many choices are there for the third node in the circuit? ________
12. Continuing this approach, how many choices are available for the fourth node in a circuit? ____
    How many choices are available for the fifth node in a circuit? _______
13. Based on the answers to questions 10, 11 and 12, how many possible circuits can be created? ________
14. Based on the answers to questions 6 and 7 above, what fraction of circuits is unique? ______
    Why does this fraction make sense? ____
    ____
15. In general, if the complete network contains '*n*' nodes, write a formula in terms of '*n*' for the total number of circuits that can be created starting at point W. ___________________
16. Recall that we discovered that some of the circuits were duplicates. Adjust the formula to account for these duplicates. _______________________________
17. Suppose there were six cities. Use the formula to calculate the number of unique circuits. _____
18. What if there were seven cities? _____
19. What if there were twenty-one cities? ____
20. What do your answers to 18 and 19 suggest about the brute force method? ________________________
    ____
21. A high-speed computer can do approximately 1 billion ($1x10^9$) computations per second. If you use the Brute Force Method, find the length of time it would take the computer to accomplish this task with 21cities.
    ____

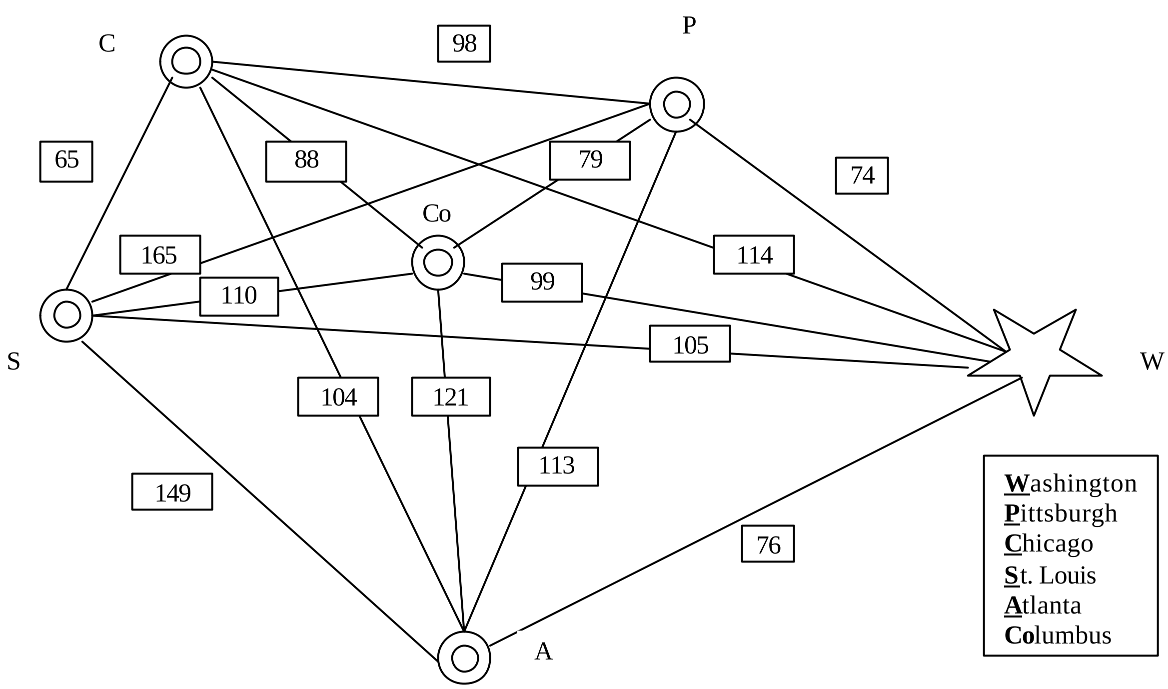
Operations researchers have developed algorithms to find low-cost routes through a network without using brute force. One method is called the "**Nearest-Neighbor Algorithm**".

Nearest - Neighbor Algorithm

1. Choose a node as your starting point.
2. From that starting node, travel to the node for which the fare is the cheapest. We call this node the "**nearest-neighbor**". If there is a tie, choose one arbitrarily.
3. Repeat the process, one node at a time, traveling to nodes that have not yet been visited. Continue this process until all nodes have been visited.
4. Complete a Hamiltonian Circuit by returning to the starting point.
5. Calculate the cost of the circuit.



A new executive just joined the firm, and she is an alumna from the Ohio State University in Columbus, Ohio. She suggested to Mr. Ira Cruit that he include a visit to Ohio State on his recruiting trip. The diagram below includes Columbus in the network, along with the associated travel costs to each of the other cities in the network. Recall that the optimal solution to the original problem involves flying first from Washington, D.C. to St. Louis.



22. Consider a minor modification to the original best route WSCPAW. Instead of flying directly from Washington to St. Louis, consider traveling first to Columbus and then on to St. Louis. What is the name of this new route? ______________________________
23. What is the cost of the new route? ______________________________
24. Use the **Nearest – Neighbor Algorithm** to find a cost efficient route for Mr. Ira Cruit's trip starting and ending in Washington, D.C. and include a visit to Columbus. ____________________
25. What is its total cost? __________
26. Why does using the **Nearest – Neighbor Algorithm** make more sense than using the **Brute Force Algorithm** in this case? ____
____

News From the World of Operations Research

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Miles traveled cut 11%.
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Sears

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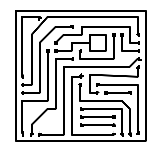
Greek Circuit Board Manufacturer: Metelco

TSP algorithm increases throughput by 10% and eliminates errors



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