Fuel Blending at Jurassic Oil: Alternate Scenario

Mr. Pete Troleum is the manager of an oil refinery for the Jurassic Oil Corporation. At the refinery, Mr. Troleum has available two grades of gasoline which must be blended at a minimum cost before they are sold. The first grade, from HyOctane, Inc., has a vapor pressure of 4.5 psi at 100°F, and contains 0.4% sulfur. The second grade, from Allif Oil, has a vapor pressure of 5.5 psi at 100°F and contains 0.25% sulfur. Jurassic Oil wants to produce a blend with the following characteristics:

1. vapor pressure of no more than 5 psi at 100° F
2. a sulfur content of no more than 0.35%.

In order to make production of the blend profitable, Jurassic Oil must produce at least 120,000 barrels (bbl) of the blend each week. They also can purchase no more than 90,000 bbl from HyOctane, Inc. each week. Mr. Troleum is able to purchase the HyOctane gasoline at $20 per bbl, and the Allif gasoline at a cost of $15 per bbl. Mr. Troleum must decide how many bbl of each grade should be used in the blend in order to minimize the cost of production.

Answering the following questions will help Mr. Troleum with his problem:

1 What decisions must Pete make? ____

2 Recommend the number of bbl of each grade you think Pete should use in the blend.

_______________ bbl of HyOctane and _______________ bbl from Allif.

3 How much would it cost to purchase the quantities you recommended?

$_______________________ for HyOctane + $_______________________ for Allif = $_______________________ total.

In this activity, we will determine the exact quantity of each grade of gasoline that minimizes the cost of producing the blend, while at the same time meeting all of the conditions for producing the blend.

Decisions, decisions, decisions …

Before we can solve Pete Troleum's problem, we must first consider the decisions that must be made. Pete must decide how many barrels of gasoline to purchase from HyOctane, and how many to purchase from Allif. We will use ***decision variables*** to represent these quantities. Every linear programming problem has a set of decision variables. Pete's problem has two decision variables, which can be represented by:

Decision variables: a set of variable quantities completely describing the decisions to be made.

A = the number of bbl of gasoline purchased from Allif
H = the number of bbl of gasoline purchased from HyOctane

4 If Pete purchases a total of 120,000 bbl of gasoline and buys as many barrels as possible from HyOctane, what would be the total cost of the purchase?

____

5 Suppose Pete purchases as many barrels of gasoline as possible from Allif. What would be the total cost of the purchase?

____

6 Write an algebraic expression to represent the purchase cost, **C**, of the blend if Pete buys **A** bbl of gasoline from Allif and

H bbl from HyOctane. ____

What's the objective?

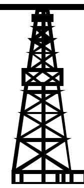
In any linear programming problem the decision maker wants to maximize or minimize some function of the decision variables. This function is called the ***objective function***. In Pete's gasoline blending problem, he wants to *minimize* the cost of buying gasoline for the blend.

Objective Function: a quantity to be maximized or minimized which is defined in terms of the decision variables.

Therefore, the objective function is: Total gasoline purchase cost, **C** = 15 **A** + 20 **H**

Now, hold on a minute …

Constraints: restrictions on the values of one or more of the decision variables.



Any restriction on one or more of the decision variables is called a ***constraint.*** Our constraints come from information in the sample problem. In all, there are four constraints. Two of them deal with properties that Jurassic Oil would like the blend to have. The others concern a limit on how much of one of the grades can be purchased each week and how much of the blend must be produced each week.

7 Write an inequality to represent the limit on how much gasoline can be purchased each week from one of the suppliers.

____

8 Write an inequality to represent the number of barrels of the blend which must be produced each week.

____

There are two remaining constraints. One concerns the vapor pressure of the blend; the other concerns the sulfur content of the blend. Representing these two constraints mathematically involves the use of a *weighted average*. Look back to your answer to number 4. In question 4, you found the total cost of purchasing 90,000 bbl of gasoline from HyOctane and 30,000 bbl from Allif.

9 What would be the *average* cost per barrel of those 120,000 barrels? ____

10 Write a sentence or two explaining why the average cost of the 120,000 bbl is *not* $17.50.

____

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Your explanation in number 10 should help you understand a formula for finding the weighted average of Allif and HyOctane:

$$\textit{average}\ \textbf{cost per bbl} = \frac{\textbf{(A) (cost per bbl for A) + (H) (cost per bbl for H)}}{\textbf{(A + H)}}$$

11 Why do we divide by **A** + **H** in the formula? ____

Try number 9 using the formula, with **A** = 30,000 and **H** = 90,000. Remember **A** costs $15 per barrel and **H** costs $20 per barrel. Did you find the same average cost per barrel? Now we are ready to use the weighted average formula to represent the vapor pressure and sulfur content constraints.

12 The vapor pressure of the blended gasoline must be no more than _______ psi . This value is a combination of a vapor pressure of _______ psi from the Allif gasoline and _______ psi from the HyOctane gasoline. Using the weighted average formula, the vapor pressure constraint can be written as the following inequality:

$$\frac{\textbf{A}\,(\quad\quad) + \textbf{H}\,(\quad\quad)}{\textbf{A} + \textbf{H}} \leq \underline{\quad\quad\quad}$$

13 Use a similar approach to write an inequality to represent the sulfur content constraint.

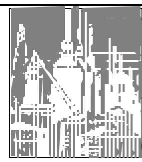
____

What *could* the solution be?

In order to solve the problem, Pete Troleum must find the amount of gasoline to purchase from each supplier which results in the lowest cost while still meeting all of the constraints. For example, Pete cannot purchase 100,000 bbl of gasoline from HyOctane.

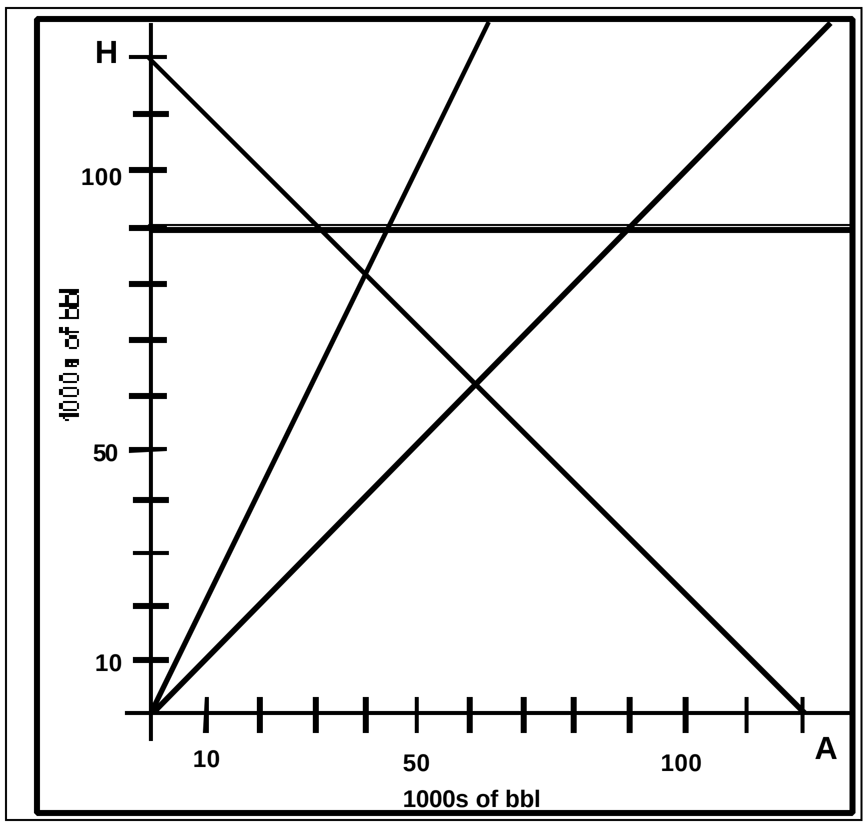
14 Explain why Pete cannot simply purchase 50,000 bbl from each of the two suppliers.

Feasible region: the set of all points which satisfy all of the constraints.



The set of all the points which *could* be the solution to the problem, because they satisfy all of the constraints, is called the ***feasible region***. One way to find the feasible region is to examine a graph showing all of the constraints.

On the graph at the right, label each line with its equation. You may find it helpful to rewrite the inequalities from numbers 12 and 13 in a standard linear form such as $\mathbf{H} \leq m\mathbf{A} + b$ or $\mathbf{H} \geq m\mathbf{A} + b$. Now shade the feasible region on the graph.



15 On the graph, locate the point which represents the recommendation you made in number 2. Is that recommendation feasible?

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But, what's the *best* solution?

The ***optimal solution*** is just the best solution to a particular problem. In this case, the values of **A** and **H** which satisfy all of the constraints and minimize the total weekly cost of purchasing gasoline is the optimal solution. The objective function, $\mathbf{C} = 15\,\mathbf{A} + 20\,\mathbf{H}$, defines the weekly cost, where **A** represents the number of barrels purchased from Allif each week, and **H** represents the number of barrels purchased from HyOctane each week. How *do* we find the lowest weekly cost that satisfies all of the constraints? We begin by finding the coordinates of each ***vertex***, or ***corner point***, of the feasible region.

16 On the graph, label each corner point with its coordinates, and write the coordinates in the spaces provided below.

(,) (,) (,) (,)

17 Next, we need to compute the cost for each corner point. Complete the table at the top of page 4 and compare the cost for each corner point.

A	H	C = 15 A + 20 H

18 Which corner point has the lowest cost? ____

19 What do the coordinates of that corner point represent? What is the minimum cost?

____

20 Is this the optimal solution? ________________

In order to be sure that the optimal solution occurs at the corner point with the lowest cost, we must convince ourselves that there is no point in the interior of the feasible region having a lower cost. Through each of the corner points on the graph of the feasible region, draw the line whose equation is 15 **A** + 20 **H** = **C**, where **C** is the cost for that corner point.

21 What do you notice about these lines? ____

22 Use these four lines and the costs they represent to make a convincing argument that there can be *no point in the interior* of the feasible region having a lower cost than the corner point with the lowest cost.

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News From the World of Operations Research

National Research Council Develops New Fertilizer Blend	Texaco Uses OMEGA System Profits up 30%	CAMP Develops Ideal Menu
LP Key to Cracking Tough Problem	Texaco Oil Company announced a 30% increase in profits – up $30 million. Sources at the petroleum giant said the increase was due to the use of OMEGA, a non-linear programming system for blending gasoline. The system, which involves 40 variables and 71 constraints, is currently in place at 7 Texaco refineries.	

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