High Step Shoes: A Linear Programming Problem

Teacher Resources

Operations Research: A Brief History

The field of Operations Research (OR) has its roots in the years just prior to World War II as the British prepared for the anticipated air war. In 1937 field tests started on what was later to be called radar. In 1938 experiments began to explore how the information provided by radar should be used to direct deployment and use of fighter planes. Until this time, the word experiment conjured up the picture of a scientist carrying out a controlled experiment in a laboratory. In contrast, the multi-disciplinary team of scientists working on this radar-fighter plane project studied the actual operating conditions of these new devices and designed experiments in the field of operations and the new term operations research was born. The team's goal was to derive an understanding of the operations of the complete system of equipment, people, and environmental conditions (e.g. weather, nighttime) and then improve upon it. Their work was an important factor in winning the Battle of Britain and operational research eventually spread to all of the military services. Several of the leaders of this effort were Nobel laureates in their original fields of study.

Their approach was later paralleled in the US, with the first team working on anti-submarine tactics. The US group developed a series of mathematical models entitled search theory that was used to develop optimal patterns of air search. Like their British counterparts, they got close to the action by riding in airplanes on patrol just as the modern operations researcher might ride in a police car or spend time in an automotive assembly plant. Currently, every branch of the military has its own operations research group that includes both military and civilian personnel. They play a key role in both long-term strategy and weapons development, as well as directing the logistics of actions such as Operation Desert Storm. In addition, the National Security Agency has its own Center for Operations Research.

OR moved into the industrial domain in the early fifties and paralleled the growth of computers as a business planning and management tool. As the field evolved, the core moved away from interdisciplinary teams to a focus on the development of mathematical models that can be used to model, improve, and even optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, simulation and decision trees. These mathematical modeling techniques form the core curriculum in masters degree and doctoral programs in operations research which can be found in either engineering or business schools. Most mathematics departments also offer one or more introductory operations research courses at the junior or senior undergraduate level.

Linear Programming: A Brief History

The first modules we developed in this series focused on linear programming. The father of linear programming is George Dantzig, who developed between 1947 and 1949 the foundation concepts for framing and solving linear programming problems. During WWII, he worked on developing various plans or proposed schedules of training, logistics supply and deployment which the military calls "programs." After the war he was challenged to find an efficient way to develop these programs. He came to recognize that the planning problem could be formulated as a system of linear inequalities. His next challenge involved the concept of a goal. At that time, when managers thought of goals, they generally meant rules of thumb for carrying out a goal. A navy man might have said our goal is to win the war and we can do that by building more battleships. Dantzig was the first to express the criterion for selecting a good or best plan as an explicit mathematical function that we now call the objective function. All of this work would have been of limited practical value without an efficient method, or algorithm, for finding the optimal solution to a set of linear inequalities that maximizes (profit) or minimizes (cost) of an objective function. He therefore proceeded to develop the simplex algorithm which efficiently solves this problem.

Economists were excited by these developments. Several attendees at a first conference entitled Activity Analysis of Production and Allocation went on to win Nobel prizes in economics with their work drawing on linear programming to model fundamental economic principles.

The first problem Dantzig solved, much to the chagrin of his wife, was a minimum cost diet problem that involved the solution of nine equations (nutrition requirements) with seventy-seven decision variables. The National Bureau of Standards supervised the solution which took 120 man days using hand-operated desk calculators. (His wife rejected the minimum cost diet as boring.) Nowadays, a standard personal computer could handle this problem in under a second. EXCEL spreadsheet software includes as a standard addition a module called “solver,” which includes a linear programming solver.

As mainframe computers became available in the 50’s and grew more and more powerful, the first major users of the simplex algorithm to solve practical problems were the petroleum and chemical industries. One use was to minimize the cost of blending gasoline to meet certain performance and content criterion. The field of linear programming grew exponentially and led to the development of non-linear programming in which inequalities and/or the objective function are non-linear functions. Another extension is called integer programming (combinatorics) in which the variables must take on only integer values. These disciplines are collectively called mathematical programming.

"Reminiscences About the Origins of Linear Programming", George B. Dantzig, Operations Research Letters, Vol. 1, No. 2, pp.43-48, 1982

Teacher Notes

Objectives:

The overall objective of this module is to motivate students to learn mathematics by demonstrating ways in which the mathematics they are learning is actually used in industry and government. Techniques of operations research such as linear programming are used to solve diverse problems in a wide variety of business and governmental settings.

This module is designed to be integrated into any first year algebra course (or the first course where students work with linear equations and inequalities in an integrated curriculum) and contains extensions which are appropriate for second year algebra students. The specific mathematics content objectives for both levels of students addressed in the module are:

- converting statements in ordinary language into the language of mathematics,
- graphing a system of linear inequalities,
- connecting the solution of a system of linear inequalities to a doable production plan, which constitutes the feasible region in this context,
- linking an objective function graphically to the feasible region in order to determine a “best” solution.
- interpreting the optimal solution.

In addition, for second year students, the extensions address the following:

- considering how real-world problems quickly grow in size by the addition of more constraints or more decision variables,
- analyzing the impact on the optimal solution of changes in the basic problem.

The High Step Shoe problem asks students to explore optimizing profits for an athletic shoe company. It is an example of what are called “product-mix” problems, and it occurs whenever a company produces more than one item. Students will learn how to apply the solution of a system of linear inequalities to solve geometrically a real-world problem involving two decision variables. One way to extend the basic problem is to consider problems involving more than two decision variables. Although the geometric technique is no longer appropriate, the problem is still solvable. If the problem can be formulated, software packages can them be used to determine the optimal solution. Thus, the “coin of the realm” today and in the future in operations research is problem formulation.

Initiating Opening Dialogue:

Ask students to consider the following questions, so as to develop a mind-set prior to using the linear programming module.

- When a company is choosing to manufacture a number of different products, what information would be necessary for the company to make a choice about how many of each product to produce?

- How could the company collect data to support its production choices?

- How often do you think the company should update its data base about markets and products? Why?

- Why might a company not always be able to produce as much of a certain product as it would like? What things would limit production?

Using the Module:

Students should read the introductory paragraphs which discuss general information about the High Step Sports Shoe Corporation. Important definitions are in boxes; these should be emphasized to students. Stress should also be placed on the importance of the constraints and how constraints impact this manufacturing industry. You may prefer to have students explore some of these questions together in groups prior to opening a whole-class discussion. For example, students working in groups might be asked to identify the variables in the problem, the quantity to be optimized (maximized in this case), and any restrictions on the variables. Whether in a group or whole class setting, students should discuss these issues before working through the lesson. You may need to question students about the restrictions (constraints). For instance, you could ask, if the goal is to maximize profit, why not just manufacture some outrageously huge number of shoes.

In discussing the formulation of this problem, the language of linear programming should be introduced and carefully defined. If students have first discussed the issues in groups, as in the foregoing example, it should not be difficult to identify the variables in the problem as *decision variables* and restrictions on the decision variables as *constraints*. Then, the quantity to be optimized should be expressed as a function of the decision variables, and this function, since it mathematically describes the basic objective in the problem, is named the *objective function*. Blackline masters for overhead transparencies are provided for each of the key definitions.

The student exploration of the basic problem is separated into four steps:

- Step 1 carries students through the definition of linear programming terms.

- Step 2 is a step-by-step development of the constraint inequalities.
- In step 3, students define the feasible region and locate its corner points.

- In step 4, students explore the feasible region to determine the optimal solution.

Solutions to all of the questions posed in the module appear at the end of this teacher's guide.

A blackline master for an overhead transparency of a graph of the feasible region is also provided. This may be used to facilitate student explorations. Students might also discuss the appropriateness of the scale.

The student activity sheets end with "News From the World of Operations Research," which are "headlines" indicating real life product-mix problems that have been solved using linear programming. In addition, one of the headlines is followed by a brief synopsis. More detailed case write-ups and related homework examples are also provided later in this teacher resource packet.

Extension: Notes

Two extensions to the basic problem are included with the materials in this resource packet. Each extension is self-contained on a single page which can be reproduced. The second extension is a continuation of the first. The extensions might be used with an Algebra 2 or higher level class. In order to develop a deeper understanding of real-life applications of mathematics, students should be allowed to discuss these extensions, as well as other sorts of situations and the different constraints they would entail.

Finding the optimal solution to the original formulation is only one step in the process of using a mathematical programming model to improve decision making. The numeric values are only estimates that may or may not be accurate and which could change with time. Consequently, mathematical modelers study how the optimal solution is influenced by key numeric values such as the amount of resource available (i.e. the value of the right hand side) or the coefficient in the objective function (i.e. profit margin of a product). There is an advanced mathematical concept called duality theory that makes it relatively easy to perform this type of analysis almost automatically. All linear programming computer packages use this theory to guide sensitivity analysis. The development and interpretation of duality theory played a significant role in the awarding of at least one Nobel prize in economics. We will introduce students to post-optimality analysis (also called sensitivity analysis) through a process of discovery. Students will be asked to change specific values and then find the new optimal solution.

Homework Problems

Three homework examples are provided. The first uses Legos to build a concrete representation of a product-mix problem. The remaining three are connected to the case write-ups which appear later in this teacher's guide and also in the "News From the World of Operations Research" section of the student text.

Projects

Two suggestions for projects are also included. The first is a short project intended to set the stage for implementing the module. Students are first asked to consider decision variables and constraints in light of their own potential life objectives. Then they are asked to do so in the context of business production. This project could comfortably be completed over a weekend.

The second project is intended as a follow-up to the use of the module. It involves actually connecting the ideas developed in the module with activities in business or government in the students' own communities.

Extension 1: Market Restrictions

High Step's arch rival, On-the-Run, Inc. recently started a major advertising campaign. As a result of this campaign High Step conducted a market study that indicated that they could not sell more than 2500 Airheads or 3000 Groundeds per week.

Write these new sales constraints as inequalities:

1. _________________________ and __________________________

On a new set of axes, graph these two new constraints together with the cutting and assembly constraints from the first part of the problem. Be sure to label each line carefully.

2. Does the solution to the original problem still work for this new problem? _________

Carefully explain your reasoning below:

The feasible region now has 4 new corner points. Fill in the table describing the corner points:

3. **Corners** **Profit**

A	**G**	**10A + 8.5G**
0	**0**	**$0.00**

4. Which corner point yields the most profit? ____________________

5. What is this profit?_________________

6. Which product (Airheads or Groundeds) should High Step advertise more heavily? Justify your answer analytically.

Extension 2: Beyond Optimality

Dr. N. Steppe is High Step's Vice President for Planning. She is trying to do two things: decrease waste and increase profits. First, Dr. N. Steppe wants to know how many hours of cutting are needed to produce the Airheads and Groundeds that correspond to the optimal strategy.

7. Use the cutting time constraint to determine the number of hours needed to produce the optimal number of each type of shoe (be sure to use the answer from number 15).

Second, Dr. N. Steppe wants to know which resources she should increase. There are many ways she could go about increasing profit. The ways that her assistants have suggested are:

* increase cutting time by running each machine for an additional 2 hours per day,
* increase assembly time by hiring 50 more workers,
* increase demand for Airheads or Groundeds by advertising more.

If Dr. N. Steppe can only do one of these options, which one should she do?

Total cutting time could be increased by running each machine longer each day. If each machine were run for an additional 2 hours per day, the total cutting time would increase from 12000 to 15000 minutes. Write a new constraint inequality for this:

8. ________________________________

9. How would this changed constraint affect the feasible region?

If 50 more workers were hired, the time available for assembly would increase from 34,000 hours to 36,000 hours. Write a new constraint for this:

10. _________________________________

11. How would this increase change the feasible region?

12. Find the new optimum solutions for each of these new problems.

Case Write Ups

Nabisco Brands

When you eat the inside of an Oreo cookie or munch on a Ritz cracker, you probably don't realize that the production of the cookie you ate was planned with mathematical programming. Production in the Biscuit Division of Nabisco involves two key operations, baking and secondary operations. In baking, raw materials are fed into an oven. Secondary operations include sorting, packaging and labeling finished products.

Scheduling and operation of bakeries is a difficult task. Each oven is able to produce many but not all products. The efficiency of the ovens varies. The secondary facilities at one site can be shared by several ovens operating at the same time. Production must be planned to keep the manufacturing and transportation costs as low as possible. The key questions that are routinely addressed with a mathematical model are:

Where should each product be produced?
How much of each product should be assigned to each oven?
From where should product be shipped to each customer?
As new products are developed where should new plants be built?
What facilities should be placed in these plants?

One interesting problem involved the study of the differences between “slug” pack vs. “dump” packs. In a traditional “dump” pack, the crackers are loose inside the box. With the slug pack, crackers are stacked in three or more columns and each column is wrapped separately. The model was used to plan the equipment changeover for different locations to convert to “slug” packaging.

A realistic problem at Nabisco could involve 150 products, 218 facilities, 10 plants, and 127 customer zones. A problem this size involves over 44,000 decision variables and almost 20,000 constraints. These problems were routinely solved in 1983 on an IBM 3033 computer in under 60 CPU seconds.

Brown, G.G., G. W. Graves and M. *D.* Honczarenko, “Design and Operation of a Multicommodity Production/Distribution System Using Primal Goal Decomposition,” Management Science, Vol. 33, No. 11 (1987) pp. 1469-1480

Agriculture in China

Both nature and economics are factors in agriculture. The availability of computers can now make agricultural planning easier and help to curb economic loss. The key question has become: If resources and equipment are fixed, can the profit from production be increased through scientific regulation and planning?

Agricultural production in Chang Qing County in China includes crop farming, livestock husbandry, forestry, fisheries, and food processing. The two most important are crop farming and livestock husbandry. While the resources and equipment available can be controlled, the weather and market prices are difficult to predict or control.

The county officials in Chang Qing used linear programming to aid the farmers in their choices of crops and other forms of agricultural production. They wished to increase the net profit while having no adverse effects on the environment. The actual problem had over 3000 variables and 100 constraints.

Using the linear programming model led to a 12% increase in crop profits and a 54% increase in animal husbandry profits. It also improved the region’s ecology and diversified the economy.

Qingzhen, Zhao, Wang Changyu, and Zhang Zhimin. "The application of Operations Research in the Optimization of Agricultural Production." *Operations Research*, Vol. 39, No. 2 (1991). pp. 194-205.

Lumber In Mexico

The use of linear programming to solve real life production problems is often hindered by two factors. One is that the terminology used in optimization problems is often mathematical in context and difficult for managers to follow and thus adopt. Secondly, management will often want an immediate answer and coming up with the correct optimization code can take time.

Linear programming was used in finding the optimal product mix in plywood manufacturing at Canadian Forest Products, Ltd. Due to the success of this company, the same idea was used in the same context at Plywood Ponderosa de Mexico, S.A.

Plywood Ponderosa de Mexico produces 85 million square feet of plywood each year. The demand for the type of wood needed is seasonal. In the winter, plywood is needed primarily for furniture manufacturing. In the summer, the construction industry is the main consumer of plywood.

Plywood Ponderosa de Mexico has 3 subsidiary companies that supply logs and there are 4 grades of logs used. A large number of different panel grades and thickness are used. This makes the product mix formulation complex. There are over 90 variables and 45 constraints.

By using linear programming, Plywood Ponderosa de Mexico found that its earlier production of thicker plywood was not the most profitable. The model showed that thinner grades gave the most profit and the company was able to increase its profits by 20%.

Ray, Asim, Emma E. DeFalomir, and Leon Lasdon. "An Optimization-Based Decision Support System for a Product Mix Problem." *Interfaces*, Vol. 12, No. 2 (1982), pp. 26-31

Homework Examples

Example 1: Lego Furniture

We can use Legos to model production in a furniture factory. The company produces only tables and chairs. A table consists of two large and two small Lego pieces, while a chair consists of one large and two small pieces. The resources available are six large and eight small pieces.

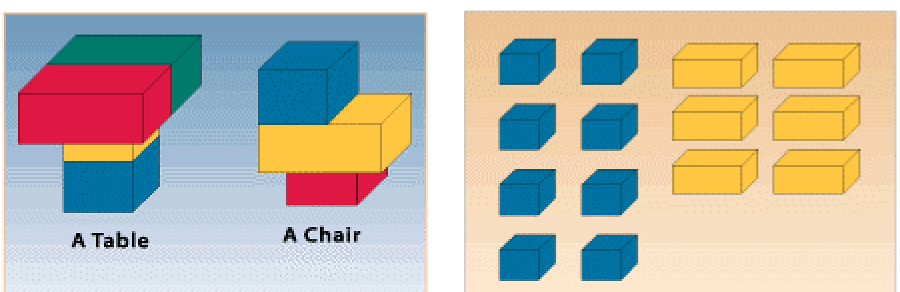


Figure 1: A table, a chair, and the available resources.

Profit for a table is $16 and for a chair is $10. Your task is to select a product mix to maximize the company's profits using the available resources.

1. Define the decision variables.

2. Use the decision variables to define an objective function.

3. Write any constraints on the decision variables.

4. Graph the feasible region.

5. Identify the corner points of the feasible region and find the optimal solution.

Lego Furniture -Teacher's Notes

If students start by trying to make as many tables as the available resources allow, they will find a solution of three tables with a profit of $48. However, making three tables does not allow them to make any chairs. This attempt at a solution is one corner point of the feasible region (3,0). Another corner point (0,4) represents making as many chairs as possible and produces a profit of $40. The true optimal solution occurs at another corner point (2,2) representing two tables, two chairs, and a profit of $52. The optimal solution in this case occurs at the intersection of the other two constraints:

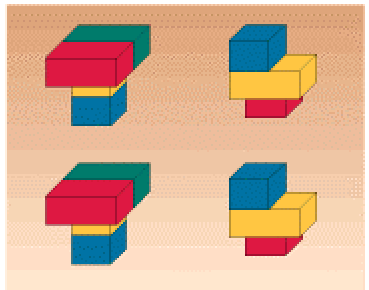


Figure 2: The Optimal Solution

Let T=number of tables and C=number of chairs.
Then 16T + 10C is the objective function to be maximized subject to the constraints:

$2T + 1C \leq 6$, because there are only six large pieces available and a table requires two large pieces, while a chair requires only one;

$2T + 2C \leq 8$, because there are only eight small pieces available and tables and chairs each require 2 small pieces; and

$T \geq 0$ and $C \geq 0$

- adapted from "Lego of My Simplex" by Norman Pendegraft appearing in ***OR/MS Today*** - February 1997 - Issues In Education: Volume 24 Number 1, p. 128;

Example 2: Not-Whole Plywood Company

In plywood manufacture, logs are cut into thin sheets called green veneer, which are dried and cut into different sizes. There are different grades of logs and different grades of veneer. Several sheets of veneer are then glued and pressed in a hot press. In the final stage, the rough plywood is sawed into an exact size and polished. The result is plywood panels of different grades and thickness.

You are the production manager of the Not-Whole Plywood Company. You have 48000 sheets of veneer grade A and 60000 sheets of Veneer grade B for use in this week's production. Not-Whole manufactures two products: Premium and Regular plywood pieces. Each piece of Premium, requires three sheets of Grade A and two sheets of Grade B and sells for $14. Each piece of Regular plywood, uses one sheet of grade A and three sheets of grade B veneer and sells for $11. Each Premium product requires 2 minutes of polishing and 40 seconds of pressing. The Regular product requires 1 minute of polishing and 30 seconds of pressing. You have 20 polishing machines and 5 presses. Each machine can be run for 40 hours during the week.

a) Formulate the production problem so as to maximize this week's revenues.
b) Determine the optimal production plan.
c) In your optimal plan are the polishing machines and presses used for all forty hours? Do you use up all of the grade A and grade B veneer?

Teacher Notes: Mini-project: gather data on plywood

Students might visit a local building supply store to gather data on prices of plywood.
Ask students to find out how the price varies by the size of the sheet and its thickness. They can then calculate whether or not the price is directly proportional to size and/or thickness. Ask students to find out how many sheets of veneer go into a piece of plywood.

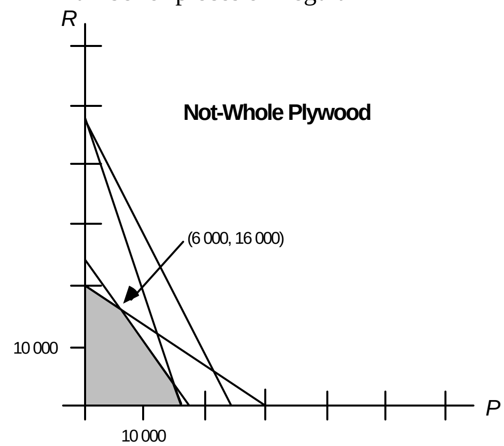
Solutions:

a) Formulation P = number of pieces of Premium and R = number of pieces of Regular
Maximize: R = 14 P + 11 R:

subject to these constraints:

$3P + 1R \le 48000$ Grade A sheets
$2P + 3R \le 60000$ Grade B sheets
$2P + 1R \le 48000$ minutes
$40P + 30R \le 720000$ seconds

$P \ge 0$ and $R \ge 0$



b) The optimal Solution: P = 6000 and R = 16000 Profit = 260,000
c) The presses are used to the maximum but not the polishing equipment. Only 28,000 minutes of polishing are used. All of the Grade B veneer is used up but not the Grade A. There are 14,000 sheets of Grade A left over with the optimal plan. The polishing resource is so large relative to the other constraints, it does not affect the feasible region.

Example 3: General Post Farms

The forecast for this year's U.S. farm production of soybeans is approximately 2.7 billion bushels and of corn is 9 billion bushels. The most recent wholesale price forecasts are soybeans at $5.20/bushel and corn at $2.50/bushel.

You are in charge of planting a 1000-acre section of your family farm and are trying to decide how many acres to plant of each crop. In your experience with your farm, an acre planted with soybeans can produce 40 bushels and an acre planted with corn produces 120 bushels. To maintain a stable income, you have signed long-term contracts with the ADP Processing Company to provide a minimum of 10,000 bushels of soybeans and 24,000 bushels of corn. Over the total season, each acre of corn requires an average of 10 hours of work and each acre of soybean requires only an average of 6 hours of labor. In total, you have only 7200 hours of available labor during the season.

a) Formulate the crop planting problem so as to maximize this season's revenues.
b) Determine the optimal planting plan.
c) How much additional revenue could be earned if there were one hundred more hours of labor available? What is the value of each additional hour? Why ?
d) Crop yields are heavily influenced by the amount of rain. The National Weather Service forecast is for a dryer than normal season. If their forecast is accurate, you are concerned that the farm yields will be down by 15% for soybean and 10% for corn. How would this change your formulation of the crop planting problem?

Teacher Notes:

Part a) The 1000 acre constraint should not be written as an equality, because there may not be enough labor hours available to work the entire 1000 acres.

Part d) Remember that this change affects not just the objective function but also the requirements to meet minimum contract commitments with ADP.

Mini-project to gather data on farming:

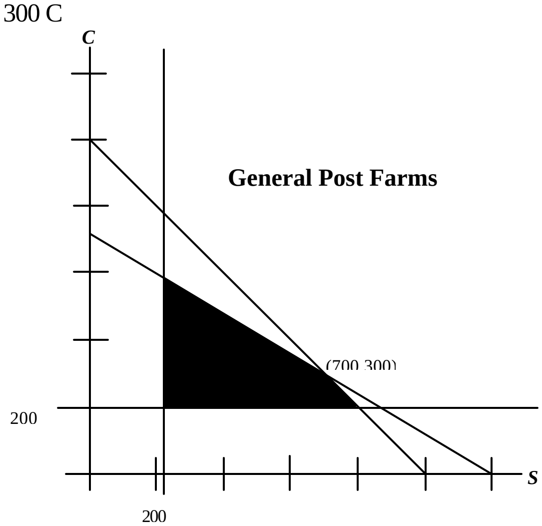
Lead a discussion of other factors that could place constraints on how much a farmer can plant and grow of each crop. These could include available equipment and water. The discussion could also explore what are the cost factors and decisions that would effect the net profit per acre and not just total revenues. These decisions could include the type of seed to use and the amount of fertilizer to use.

A key concept in farming is that there is significant uncertainty regarding price at harvest and yields on the particular farm. The harvest price is affected by national and international trends in planting and weather. A particular farm's yield is influenced by local weather conditions. You might ask students to gather data on variations in yield from one region to another and from one year to another. You may also ask students to gather more information regarding the different production costs associated with running a farm. This task can be easily accomplished by a Web search. For your information Soybean Production Costs and Corn Production Costs will lead students directly to spreadsheets with details on the cost per planted acre in different parts of the US. Teams of students could also be asked to explore a global perspective by picking a particular country and tracking down similar data for that country.

Solutions:
a) Formulation
Maximize $P = (5.2)40\,S + (2.5)120\,C = 208\,S + 300\,C$

subject to these constraints:

$S + C \leq 1000$
$40S \geq 10000$ bushels
$120C \geq 24000$ bushels
$6S + 10C \leq 7200$ hours
$S \geq 0 \quad C \geq 0$



b) Optimal Plan: S = 700 and C =300 Total revenue = $235,600

c) The added value is $2300. S = 675 C = 325 Total revenue = $237,900.
Each hour of the 100 hours is worth $23.

d) New formulation

Maximize P = 176.8 S + 270 C,

subject to constraints:

$S + C \leq 1000$

$34S \geq 10000$ bushels

$108C \geq 24000$ bushels

$6S + 9C \leq 7200$ hours $S \geq 0$ $C \geq 0$

Projects

I. Allocate Study Time for Final Exams: This project might serve to introduce some of the conceptual issues in the module in a familiar context. It could easily be done in class.

Overview:

The allocation-of-time (a limited resource) during final exam week fits the generic framework of mathematical programming. This topic can be used for an open-ended discussion to explore the concepts of decision variables, constraints, objectives and non-linearity.

Questions to Consider:

Final Exams are coming up soon and you are trying to make plans to study.

1. What decision variables are under your control? Write a statement in words that represents a decision variable?

2. What constraints do you face? Are they all less than or equal to constraints? or are some of them greater than or equal to constraints?

3. What is your ultimate objective that will guide your decisions? Are you trying to maximize something or minimize something? How would your grade point average going into the final exam affect your decisions?

Notes to Teachers:

1. The decision variables could be the number of hours during the week to study for each course, or more specifically, the number of hours to study on Monday, Tuesday,.. for each course.

2. Constraints might include amount of time available to study during the week or each day, or the minimum number of hours of study for each course so as to obtain a grade of at least …. How would the students estimate the value of this minimum? Will it differ for each course? You might also ask how having a study partner could complicate the problem?

3. Students should be questioned about their objective. Is the objective to maximize overall grade point average subject to constraints on achieving a minimum grade on each course? Or is the goal to minimize the amount of time spent studying while achieving minimum grades on each course?

   You may also want to ask students to write an objective function. In doing so, they will need to think about whether each hour of studying in a particular course adds a given number of points to the estimated final exam score? In upper level classes, you might also ask students if they think the final exam score is a linear function of the number of hours spent studying? They could also consider whether letter grades or numerical grades would make any difference in the formulation.

II. Interview Manager: This project is intended as a possible follow-up to the study of the module. In the High Step Shoe Corporation case, we introduced new terminology with regard to

- decision variables,
- constraints, and
- the objective function.

In creating the entire linear programming formulation there are a set of numeric values that are crucial to the formulation. In the real-world applications a significant amount of effort is often required to determine these values.

Numeric multipliers (coefficients) in the objective function. In the High Step Shoe Corporation case, the objective was to maximize profit. We simply stated that each Airhead produced a profit of $10. It is not always an easy task for a company to determine the profit margin it makes on each product. It is often easier to determine the gross revenues rather than profits by simply using the sale price and multiplying it by the number sold.

Numeric multipliers on the left hand side of the equation. In the High Step case, the cutting time constraint and the assembly constraint both had different coefficients for the two products. These numeric values represent the amount of each resource that is used up by the production of one Airhead or one Grounded. To determine this numeric value would require extensive data collection and analysis. In addition, through training or increased experience, these coefficients could change over time as workers become more efficient at their tasks. Does the coefficient go up or down as the workers become more efficient?

Numeric value of the right hand side. The numeric value on the right hand side of the cutting and assembly constraints represent the amount of a limited resource available for production. In the case of the machine, it may be difficult to change this value without spending a lot of money. Consequently, in formulating the problem, the number of machines was taken as a given and not necessarily a variable under the control of the production manager. In the extensions we did discuss the possibility of running the machines for more hours.

The second resource was the number of workers. Again, this might not be a decision variable (e.g., there could be a freeze on hiring new workers). In other contexts, the number of workers could also have been a decision variable. In the extension we asked students to explore the value of having more workers.

In formulating any mathematical model, the model builder must decide what variable he or she wants to focus on and which variables or resources, for now, are to be considered fixed and not subject to change or management control.

As students approach the open-ended project, they will need to think not only in broad terms about decision variables, constraints and the objective function, but also about how to determine the numeric values that are needed to construct the objective function and to write the constraints.

Overview:

All businesses have goals. These same businesses also have obstacles that must be surpassed if the goals are to be achieved. The purpose of this project is to study a business and explore the ways in which mathematics might be used to achieve the goals of the manager and owner.

Preparation:

With the other members of your group, schedule an appointment to interview a manager or owner of a business in your neighborhood or near your school. Almost any business will be a good source of information for the purposes of this project. Restaurants, dry cleaners, police forces, and many other diverse types of businesses and agencies each have their own set of goals and constraints.

Data Collection:

Here are some questions that you should be sure to ask:

- What are the goals of the business?

- What constraints hinder the achievement of the goals?
    1. personnel?

2. demand?
3. other factors?

- What decisions can a manager or owner control?
- What are the company's costs and revenues?
- What technological or mathematical tools help achieve the goals of the company?
- What tools does the company not use that might help?

The Report:

The members of the group should take the information gathered in the interview and write a report that includes the following information:

Interview Experience:

Outline the communications skills that were used and describe the interview itself. How was the interview set up? Where did it take place? How was information recorded accurately and verified? What languages were used to conduct the interview?

Problem Formulation:

Can you define one or more decision variables under the management's control. Do you understand the goals of the business and how to tell if the business is achieving its goals? Can you relate the decision variables to the goals? Could you formulate the goals in mathematical terms? Do you understand the constraints on the business? Could you formulate the constraints in mathematical terms? If you could create the mathematical formulations but don't have enough information, how could you get the information?

Recommendations:

On the basis of your analysis of the data you gathered, what recommendations could you make to the manager or owner of the business that would help them achieve their goals for the business? The recommendations might be in the form of specific changes in the decisions that are made or might suggest further evaluation or data collection. Any recommendations should be substantiated by the data you collected and subsequent analysis.

Solutions to Base Module and Extensions

High Step Shoes Module

1. $P = 10A + 8.5\,G$
2. 6 X 50 X 40 = 12000
3. Machines can only work 12000 minutes per week, never more.
4. 850 X 40 = 34,000
5. $7A + 8G \leq 34{,}000$
6. You cannot make a negative number of shoes, but it is possible to make zero shoes.
7. (0,4250) (0,0) (4000,0) (2800, 1800)
8. answers vary
9. 10(0) + 8.5(4250) = 36,125
10. (4000) + 8.5(0) = 40,000
    10(2800) + 8.5(1800) = 43,300
10. (2800, 1800)
11. 10/8.5 or 100/85 , They are parallel
    Yes, 25,000, 30,000 and 40,000
    At the corner (2800, 1800)

Extension 1: Market Restrictions

1. $A \leq 2500$ and $G \leq 3000$
2. No, it is outside the feasible region
3.

A	*G*	$10A + 8.5G$
0	0	$0.00
0	3000	$25,500
1429	3000	$39,790
2500	2062	$42,527
2500	0	$25,000

4. (2062)
5. $42,527
6. Airheads. When Airhead production is increased while Groundeds are decreased, profit goes up. Airheads make more money and take less time for a worker to put together.

Extension 2: Beyond Optimality

7. hours
8. $3A + 2G \leq 15000$
9. It makes it larger, higher on the y axis.
10. $7A + 8G \leq 36000$
11. It makes it larger also
12. Produce 2500 Airheads and 2313 Groundeds for a profit of $44,600.