High Step Sport Shoes: A Linear Programming Problem

Linear programming is a method often used to solve large, complicated problems. These problems often require a manager to determine how to use the company's limited resources most efficiently. In order to see how linear programming is used to solve such problems, let's investigate a problem experienced at the High Step Shoe factory. The High Step Shoe problem is just a fraction of the size of those actually encountered in the real world.

Step 1: Understanding the problem

Mr. B. Ball, director of manufacturing for the High Step Sports Shoe Corporation wants to maximize the company's profits. The company makes two brands of sport shoe, *Airheads* and *Groundeds*. The company earns $10 profit on each pair of *Airheads* and $8.50 profit on each pair of *Groundeds*. The steps in manufacturing the shoes include cutting the materials on a machine and having workers assemble the pieces into shoes.

? **<u>Decision variables</u>** represent a quantity that a manager can change. Example: the number of shoes to be made.

***A* = number of pairs of *Airheads* produced each week**
***G* = number of pairs of *Groundeds* produced each week**

Mr. B. Ball's goal is to make the most money or **maximize** his profits. Using ***A*** and ***G***, write a function to model the profit.

(1.) Profit = _______ ***A*** + _______ ***G***

This function is called the **objective function.**

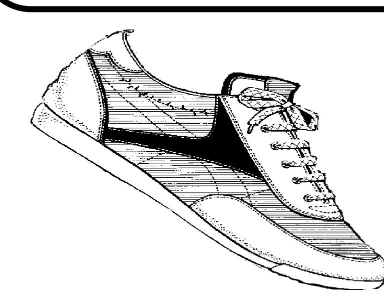
<u>Objective function</u>

the equation that represents the goal of either maximizing profit or minimizing cost

The number of machines, workers, and factory operating hours put **constraints** on the number of pairs of shoes that the company can make. High Step Sport Shoe Corporation has the following constraints. There are *6 machines* that cut the materials, *850 workers* that assemble the shoes, and the assembly plant works a *40 hour* week.

<u>Constraints</u>

are limitations created by scarce resources (time, equipment, etc.). They are expressed algebraically by inequalities.



Step 2: A System of Inequalities: Constraints

Machine Cutting Constraint Inequality

Each hour, each cutting machine can do 50 minutes of work. How many minutes of work can 6 machines do in a 40 hour work week?

(2.) _____ machines x ______ min/hr x _______hr/wk = __________

Each pair of *Airheads* requires 3 minutes of cutting time while *Groundeds* require 2 minutes.
The mathematical model representing the constraint of the cutting machines is **3*A* + 2*G* £ 12000 minutes.**

(3.) Why do we use £ 12000 minutes instead of = 12000 minutes?

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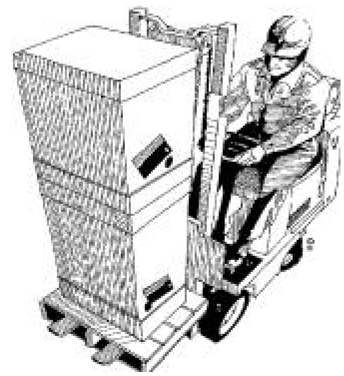


Worker Assembly Constraint Inequality

4. How many minutes of work can 850 assembly workers do in a week?

_____ workers x ______ hr/wk each = ___________ hr/wk total.

Each worker takes 7 hours to assemble a pair of *Airheads* and 8 hours to assemble a pair of *Groundeds*. Write the **constraint inequality** modeling the amount of time it takes to assemble the shoes for the week.

5. **__________*A* + __________*G*** $\le$ **__________ hours**

Other Constraints

The number of pairs of shoes that High Step manufactures is never negative, but could possibly be zero. Why?

6. ____

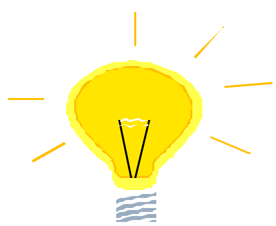
Therefore, two more constraints on our variables, *A* and *G* are, $A \ge 0$ **and** $G \ge 0$.

Step 3: Graphing a System of Inequalities - Feasible Region

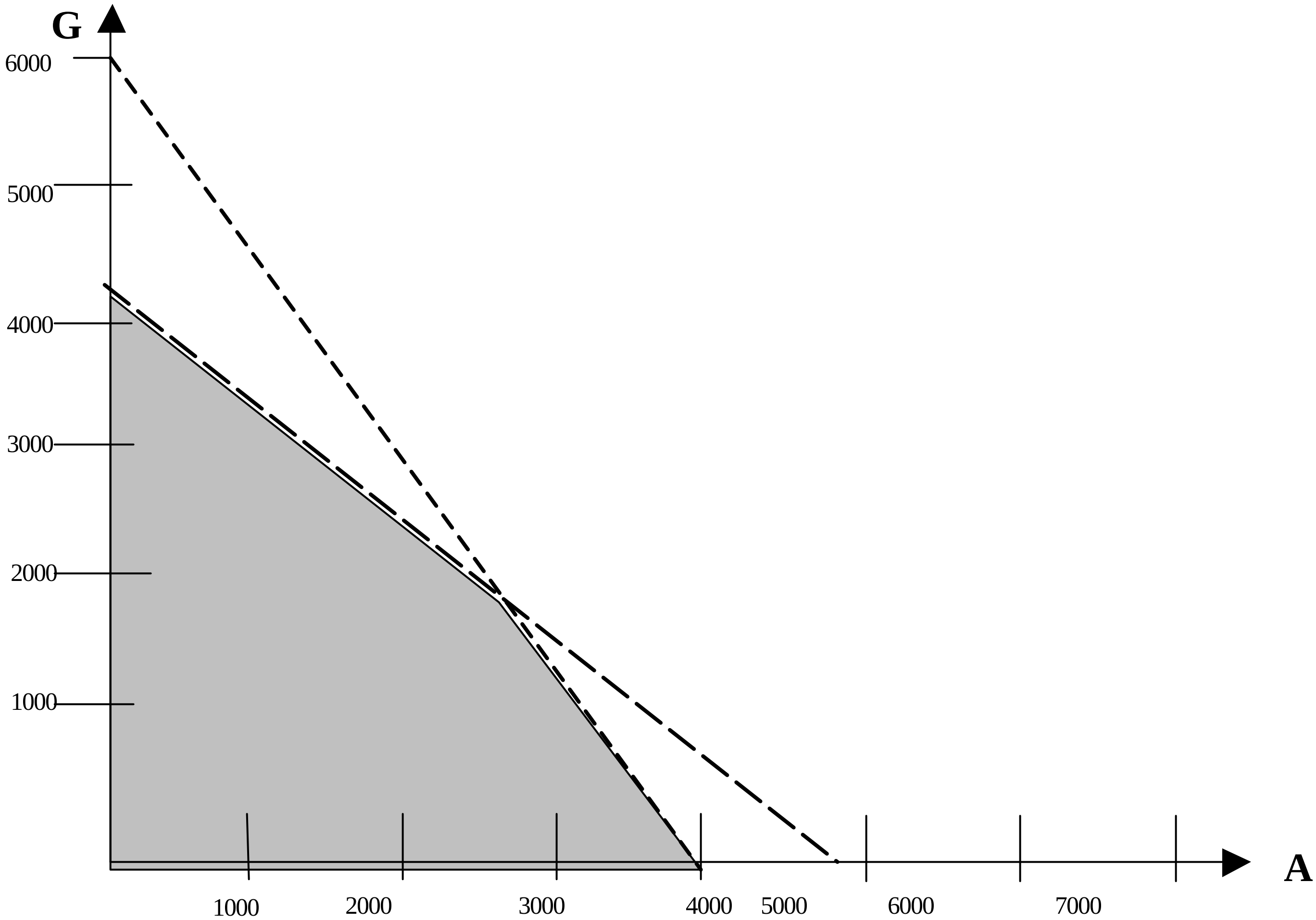
Our system of constraint inequalities is:

1) $3A + 2G \le 12000$
2) $7A + 8G \le 34000$
3) $A \ge 0$
4) $G \ge 0$.

The graph of this system of constraint inequalities appears at the top of the following page. The shaded region represents the set of points which satisfy all of the constraints. Values of *A* and *G* which satisfy all of the constraint inequalities are called **feasible** and the set of all such feasible points is called the **feasible region.**



All possible solutions to the problem lie in the feasible region or on the boundarv



7. A. Label each line with its equation.
   B. Label all points of intersection by solving the system of inequalities. These intersections are called **corner points**.

Step 4: Best Production Plan: Searching for the Optimal Solution

The best solution for this problem gives the High Step Sport Shoe Company a maximum profit. The process of determining this best solution is called **optimizing**, and the solution itself is called the **optimal solution**.

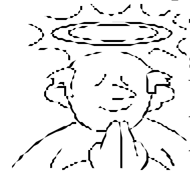
To determine the optimal solution, there are many strategies you could use.

8. One way is to try all the possible answers.

- Pick three points inside the feasible region, note them in the table.
- Test your points in the Profit equation.

A = Airheads	G = Groundeds	Profit = 10A + 8.5 G
ex. 1000	3000	35500 = 10(1000) + 8.5(3000)

Optimum



The optimal solution yields the best solution (e.g. the most profit or the least cost)

Compare your answers with other students and see who has the most profit.

9. Test all four corner points.

A = Airheads	G = Groundeds	Profit = 10A + 8.5G
0	0	0 = 10(0) + 8.5(0)

The Corner Principle

states that the optimal solution will always lie on a corner of the feasible

10. Which point gives you the largest profit from both tables?

____

This is an example of the **corner principle.**

Let's investigate why the corner principle works. The graph of the feasible region is provided on the next page.

Let's turn the profit equation around to yield 10A + 8.5G = P, and substitute different quantities for the profit (like $25,000, $30,000, $40,000, $45,000, and $50,000) to see just how much money we can make for the High Step Company.

$10A + 8.5G = 25{,}000$ $10A + 8.5G = 30{,}000$ $10A + 8.5G = 40{,}000$ $10A + 8.5G = 45{,}000$ $10A + 8.5G = 50{,}000$

11. Solve each equation for ***G***. What is the slope of each line?______________ Draw each line on the graph of the feasible region on the previous page. What do you notice about all of these lines? ____

Are any of these values of *P* feasible? ________ Where will the line representing the **optimal solution** intersect the **feasible region**? ____

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