Hot Dog Sales at Frankfurter High – How Many Should We Order?

Felix J. Frankfurter High School in New York City has a nationally ranked basketball team that regularly makes it to the state championship of New York. They play ten home games each season and every game is sold out. The sports booster club, The Frankfurter "Hot Dogs", sells hot dogs for $2.00 apiece during games. They buy hot dogs and rolls by the dozen. The cost of a dozen hot dogs is $6.56 and of a dozen rolls is $1.44. A local restaurant donates the condiments. At the end of every game, all unsold hot dogs and rolls are donated to the local homeless shelter.

1. If the club sells a dozen hot dogs, how much money would they take in? ______________________
2. How much profit would they make? _____________________________________

The Frankfurter "Hot Dogs" were optimistic about sales at the opening home game and ordered 16 dozen. However, they were only able to sell 9 dozen.

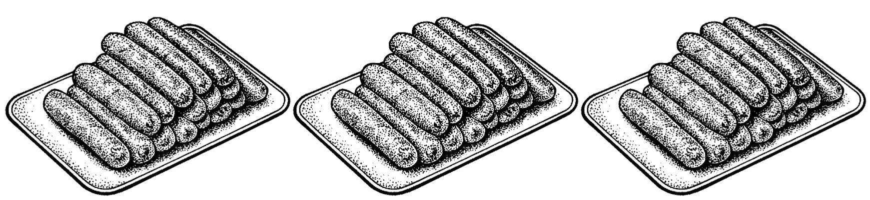
3. How much net profit did they make at the first game? _________________
4. How much net profit would they have made if they had ordered exactly nine dozen hot dogs and rolls? _________________
5. How much lost profit resulted from over-ordering? ___________________
6. What conclusion might you draw about future purchases? ____

Having been stuck with leftovers at the previous game, the Frankfurter "Hot Dogs" decide to order only nine dozen hot dogs and rolls for the next game. This game drew a large hungry crowd and the game turned out to be a boring runaway. They ran out of hot dogs early in the third quarter and the crowd was clamoring for more! They estimated they could have sold another five dozen hot dogs.

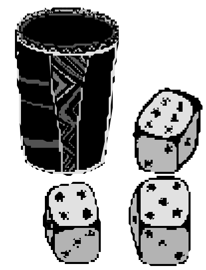
7. How much profit did the booster club make at the second game? _________________
8. How much profit would they have made if they had ordered and sold another five dozen? ________
9. How much potential profit was lost due to under-ordering? ___________________
10. What conclusion might you now draw about future purchases? ________________________
11. What factors might you consider before placing your next order? _________________________
12. How many dozen would you choose to order for the next game? __________________________
13. How much confidence do you have in your choice? Why? ___________________________________

The members of the booster club are frustrated by the situation. It seems they simply can't predict *exactly* how many hot dogs they will be able to sell at any specific home game. They realize that the reason they can't make an accurate prediction is that the demand for hot dogs is random. (The uncertainty of demand is a common problem that companies face when deciding how much to order or produce.) Mrs. Matrix, their math teacher/sponsor, suggests looking at the booster club's records over the past five years. They learn that

- they always ordered 16 dozen hot dogs and rolls,
- the average demand was for around 10 or 11 dozen hotdogs,
- the most they ever sold was 16 dozen,
- the least was 5 dozen,
- the demand for hot dogs frequently was between 9 and 12 dozen, and
- the demand was rarely less than 9 dozen or more than 12 dozen.



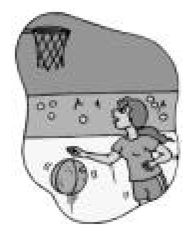
They realize that there must be a better way to decide how much to buy in the face of this uncertainty. Mrs. Matrix suggests creating a simulation to model the problem. This would be a practical way to see costs and profits over a period of time without spending any money. To create the simulation, they decide to use three number cubes having six faces numbered 1 through 6. The sum of the numbers will always be a whole number between 3 and 18, inclusive. This models the range of values for hot dogs purchased at their basketball games. They wonder if the average result of rolling three number cubes will accurately model their average demand for hot dogs. They realize that since each of the six numbers on each number cube is equally likely to occur, the long-term average for a single cube is just the average of the six numbers.



14. What is the long-term average of rolling a single number cube with six faces numbered from 1 through 6? ___
15. What is the long-term average of rolling three of these number cubes? ____
16. Explain why rolling three number cubes does or does not accurately model the demand for hot dogs at Frankfurter High's home basketball games. ____

____

To simulate the demand for hot dogs at Frankfurter High's home basketball games, you will work with a partner and a set of three number cubes. Each roll of the set of three number cubes will be used to represent the demand in dozens of hot dogs for one home game. You will simulate the demand for hot dogs for one season by rolling the number cubes ten times and recording each sum. Finally, you will compute total sales and profit (or loss) for each game, as well as totals and averages for a season.



17. With your partner, decide how many dozen hot dogs and rolls to order every week. ______
18. Calculate the total weekly fixed cost for the number of hot dogs and rolls you decided to order. Enter that value here _________ and on each row of the column labeled "Weekly Fixed Cost" in the table below.

In the table below, record the sum of the three number cubes in the column labeled "random demand". Compare the demand to your weekly order and record the number of dozens of hot dogs sold. Then for each week, compute the total revenue and profit and enter those values in the table. Compute the totals for each column, as well as their averages, and enter those values in the table. Observe and record the minimum and maximum values in each column. Complete the last three columns for the old policy of always ordering 16 dozen at a cost of $128 per game.

	Your New Policy: Always Order _________ dozen					***Old Policy: Always Order 16 Dozen***		
Week	***Random Demand*** (dozens)	***Hot Dogs Sold*** (dozens)	***Total Revenue***	***Weekly Fixed Cost***	***Profit =*** *Revenue - Cost*	***Hot Dogs Sold*** (dozens)	***Total Revenue***	***Profit =*** *Revenue - $128*
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
Totals								
Average								

19. What was the least profit you made for any game? ___________
20. What was the most profit you made for any game? ___________
21. What caused this difference? ____

22. How does the total profit for the season under the old policy compare to the total profit under your new policy?
____

This activity simulated one representative season. Do you think that every simulation would produce approximately the same results? Let's explore this issue by conducting another simulation of one season with the same ordering policy used above. Perform the same procedures to carry out the simulation a second time. Use the following table to record the sample results from the second simulation and the associated calculations.

	Your New Policy: Always Order _________ dozen					***Old Policy: Always Order 16 Dozen***		
Week	***Random Demand*** (dozens)	***Hot Dogs Sold*** (dozens)	***Total Revenue***	***Weekly Fixed Cost***	***Profit =*** *Revenue - Cost*	***Hot Dogs Sold*** (dozens)	***Total Revenue***	***Profit =*** *Revenue - $128*
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
Totals								
Average								

23. What was the least profit you made for any game? ______
What was the most profit you made for any game? ______
24. How does the total profit for the season under the old policy compare to the total profit under your new policy?
____
25. Compare the results of the two simulations. Are they identical? _______________
Why or why not? ____

Next, for each demand, we will use the relative frequency to estimate the probability.
26. What does relative frequency mean?____

Recall that the possible outcomes simulated by tossing three number cubes are the integers from three to eighteen. For each of your two simulations, record the frequency of each demand possibility in the table below. Calculate the relative frequencies by dividing each frequency by ten. Finally we will pool all of the sample results from the simulations in the class. Record the pooled results for the whole class on the right hand side of the table.

Demand Values	**First Individual Frequency (*i*)**	**First Relative Frequency (*i/10*)**	**Second Individual Frequency (*j*)**	**Second Relative Frequency (*i/10*)**	**Class-Pooled Frequency (*p*)**	**Class -Pooled Relative Frequency**
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
Min						
Max						

Using your graphing calculator, create histograms of the two individual relative frequencies and of the pooled relative frequency.

27. Describe and discuss the shape of each of the three histograms.
____
____
____

28. Are any of the histograms symmetrical? If so, which histogram is most symmetrical?
____
Why does this make sense? ____
29. Which value occurred most frequently in the individual results? ______ _____ Pooled results? _____

As a class, choose any one of the possible demand numbers, and record that choice here: _____

30. For your individual simulations, what is the relative frequency of the demand number chosen above? _____
31. For the class-pooled simulations, what is the relative frequency of that demand number? ______
32. In your class, identify the individual who recorded the *largest* relative frequency of the demand number chosen by the class; what is the relative frequency of that number? _____ Next, identify the individual who recorded the *smallest* relative frequency of that demand number; what is that smallest relative frequency? ______
33. What is the order relationship of the three relative frequencies from questions 31 and 32?
____
34. Write an inequality to illustrate this order relationship. ____
35. Based on the individual simulations of 10 games, which ordering policy resulted in the most profit? ________
36. What was the total profit? _____________

Finally, we will determine the best long-term ordering policy by using the pooled relative frequency data to simulate long-term random demand. In order to evaluate every possible ordering policy, your teacher will assign to you an ordering policy somewhere between 3 and 18 dozen, inclusive. Enter your assigned policy in the blank below. Complete the first four columns of your table. The pooled relative frequency for each demand in the simulations approximates the proportion of games having that demand in the long-term. Multiplying the profit by the relative frequency and summing that column determines a weighted average profit per game under your assigned policy.

Your Assigned Policy: Always Order _________ dozen

Demand	***Hot dogs sold***	***Total revenue***	***Profit***	***Pooled Relative Frequency***	***Profit X Pooled Relative Frequency***
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
				Weighted Average = (Column Sum)	

39. Compare the weighted average for each possible policy to determine the best ordering policy and its associated weighted average profit.
Best Ordering Policy __________ Weighted Average Profit __________
40. Compare your answers in 37 and 38 to your answers in 39 and explain any differences.
____
____

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