

**EXTENSION 1:** In order to gain a better understanding of the Hot Dog Sales simulation, in this extension, we will analyze the entire probability space for tossing three number cubes.

1. When *one* number cube is tossed, how many outcomes are possible? \_\_\_\_\_  
When *two* are tossed? \_\_\_\_\_ When *three* are tossed? \_\_\_\_\_

Now suppose we begin to think about what some of the outcomes for three number cubes look like.

2. Write one of the possible outcomes as an ordered triple  $(x, y, z)$ . \_\_\_\_\_ Write another possible outcome in the same form. \_\_\_\_\_ The entire probability space could be written in this way, if we could be assured of not missing any possibility. Can you think of a *systematic* way of writing all of the possible outcomes?

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3. How many ordered triples in the probability space have the form  $(1, 1, z)$ ? \_\_\_\_\_ List them all and next to each one, write the *total* of the three number cubes.

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4. Repeat step 3 for all of the ordered triples of the form  $(1, 2, z)$ . Then do the same for ordered triples of the form  $(1, 3, z)$  through  $(6, 6, z)$ .

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5. For how many of the ordered pairs from steps 3 and 4 is the *total* of the number cubes 3? \_\_\_\_\_
6. For how many of the ordered pairs from steps 3 and 4 is the *total* 18? \_\_\_\_\_
7. What is the probability of obtaining a *total* of 3? \_\_\_\_\_ ... of 18? \_\_\_\_\_
8. In a similar manner, find the probabilities of obtaining *totals* of 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17.

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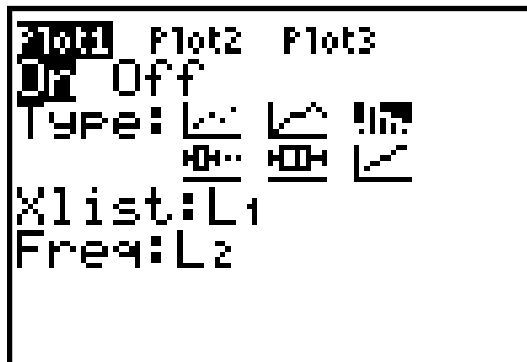
9. Convert each of the probabilities from steps 7 and 8 to decimal form. How do these probabilities compare to the pooled relative frequencies from the Hot Dog Sales activity?

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**EXTENSION 2:** The programs below were written for a TI-83 calculator. They can be used to simulate rolling three number cubes 100 times, recording the data, and drawing a histogram of the simulated data.

PROGRAM:NEWSBOY :Lbl 0 :16→dim(L1) :16→dim(L2) :Fill(0,L2) :For(T,3,18,1) :T→L1(T-2) :End :For(T,1,100,1) :prgmCUBE :If S=3:1+L2(1) →L2(1) :If S=4:1+L2(2) →L2(2) :If S=5:1+L2(3) →L2(3) :If S=6:1+L2(4) →L2(4) :If S=7:1+L2(5) →L2(5) :If S=8:1+L2(6) →L2(6) :If S=9:1+L2(7) →L2(7) :If S=10:1+L2(8) →L2(8) :If S=11:1+L2(9) →L2(9) :If S=12:1+L2(10) →L2(10) :If S=13:1+L2(11) →L2(11)	:If S=14:1+L2(12) →L2(12) :If S=15:1+L2(13) →L2(13) :If S=16:1+L2(14) →L2(14) :If S=17:1+L2(15) →L2(15) :If S=18:1+L2(16) →L2(16) :End :max(L2)+1→Ymax :0→Xmin:0→Ymin :20→Xmax:1→Xscl :DispGraph  PROGRAM:CUBE :Lbl 0 :0→S:0→Q :Lbl 1 :S+randInt(1,6)→S :1+Q→Q :If Q=3 :Return :Goto 1
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Copy each program into your calculator. Then, use “STAT PLOT” to set up Plot1 as shown in the following screen images. Finally, execute the program named “NEWSBOY.”



1. What do you observe about the histogram? \_\_\_\_\_

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Use “Prgm EDIT” to change the 8<sup>th</sup> line of the program so that it reads :FOR (T, 1, 500, 1), and execute the new program. (This program will take about 3 minutes to execute.)

2. How many rolls of the number cubes is the new program simulating? \_\_\_\_\_
3. How does the histogram of this simulation compare to the first one? \_\_\_\_\_

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Now, edit the “NEWSBOY” program so that it simulates 1000 rolls of the number cubes, execute the program, and compare the histogram to the previous two. (Be patient! It will take about 6 minutes to execute the program this time.)

4. What observation(s) can you make about the histogram as the number of rolls of the number cubes increases? \_\_\_\_\_

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**EXTENSION 3:** This extension involves the use of a graphing calculator and is written for a TI-83 calculator.

Using “STAT EDIT,” enter the integers 3 through 18, inclusive as the elements of L1 (i.e.,  $L1(1)=3$ ,  $L1(2)=4$ , ... ,  $L1(16)=18$ ). Then, using the results of EXTENSION 1, enter the probabilities of obtaining 3, 4, ... , 18 as the elements of L2 (i.e.,  $L2(1)=1/216$ ,  $L2(2)=3/216$ , ... ,  $L2(16)=1/216$ ). Finally, draw a histogram using L1 as the x-list and L2 as the frequency list. Set the viewing window to [0,20] by [0,0.2] and be sure to set  $XSC1=1$ .

1. What do you observe about the histogram? \_\_\_\_\_  
\_\_\_\_\_
2. How does this histogram compare to the histograms from the Student Activity or EXTENSION 2? \_\_\_\_\_  
\_\_\_\_\_

Next, paste `normalpdf(X-0.5,10.5,2.96)` into Y1. (You can access `normalpdf` (by pressing 2nd VARS. It is the first item in the DISTR submenu.) Turn the STAT PLOT off, and graph Y1.

3. Describe the graph. \_\_\_\_\_  
\_\_\_\_\_

Now, graph Y1 and the histogram from #1 in the same viewing window.

4. What do you observe about these two graphs? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Homework**

**1. Salvage Value**

The unsold hot dogs and buns will be sold at a steep discount to fans as they leave. Assume that you will be able to sell all of your leftover dozens at a price of \$5 per dozen. Use the tables below to reevaluate the policy of ordering 11, 12, or 13 dozen with this “salvage” value of \$5 per dozen. For the column “Pooled Relative Frequency,” use the data generated earlier in class. For example, if you ordered 11 dozen and the demand was only nine dozen, there would be two dozen to sell at a discount. The total net revenue would increase by  $2(\$5) = \$10$ .

After completing the tables below discuss the affect of salvage value on each ordering policy.

What was the best policy with no salvage value? \_\_\_\_\_

What was the best policy with a \$5 salvage value? \_\_\_\_\_

<b>Always Order <u>11</u> dozen</b>								
		<i>No Salvage Value</i>		<i>\$5/dozen Salvage Value</i>			<i>No Salvage Value</i>	<i>\$5/dozen Salvage Value</i>
<i>Demand</i>	<i>Hot dogs sold</i>	<i>Total revenue</i>	<i>Profit A</i>	<i>Total revenue</i>	<i>Profit B</i>	<i>Pooled Relative Freq.</i>	<i>Profit A X Pooled Relative Frequency</i>	<i>Profit B X Pooled Relative Frequency</i>
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
<b>Weighted Average = (Each Column Sum)</b>								

Always Order <u>12</u> dozen									
Demand	Hot dogs sold	No Salvage Value		\$5/dozen Salvage Value		Pooled Relative Freq.	No Salvage Value		
		Total revenue	Profit A	Total revenue	Profit B		Profit A X Pooled Relative Frequency	Profit B X Pooled Relative Frequency	
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
<b>Weighted Average = (Each Column Sum)</b>									

Always Order <u>13</u> dozen									
Demand	Hot dogs sold	No Salvage Value		\$5/dozen Salvage Value		Pooled Relative Freq.	No Salvage Value		
		Total revenue	Profit A	Total revenue	Profit B		Profit A X Pooled Relative Frequency	Profit B X Pooled Relative Frequency	
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
<b>Weighted Average = (Each Column Sum)</b>									

## 2. Airline Overbooking

The Big Sky Airline is active during summer season. It flies a ten-seat plane primarily between major cities and resort towns. The cost of a one-way unrestricted ticket is \$100. (An unrestricted ticket means a “no show” gets his money back.) Historically, 20% of the people with reservations are “no shows.” It is therefore evaluating its policy on overbooking during peak periods. By federal regulation, any person who has a confirmed reservation but cannot be seated and must wait for the next flight is to be compensated with \$200. For example, if eleven tickets were sold and all of the travelers showed up, Big Sky will take in \$1100 and then pay out \$200 for a net of \$900. The probability distribution of the number of people who show up at the airport follows the Binomial Distribution with  $p = 0.8$  and  $n =$  number of reservations.

Let:  $X =$  number of customers who show up.  
 $R(X) =$  Gross revenue for  $X$ .  
 $C(X) =$  Cost of overbooking.  
 $P(X) =$  probability that  $X$  customers will show up. (This is based on the Binomial Distribution.)  
 $n =$  maximum number of reservations accepted. (This is the policy to be set.)

The expected net revenue (average gross revenue minus the average cost of overbooking) is calculated using the formulas below.

$$E(R(X)) = \sum R(X) * P(X) \text{ Summed over } X = 1 \text{ to } n.$$

$$E(C(X)) = \sum C(X) * P(X) \text{ Summed over } X = 1 \text{ to } n.$$

Explain why “NA” appears in specific boxes below. \_\_\_\_\_

Explain why  $E(C(X))$  for  $n = 10$  is always zero. \_\_\_\_\_

Calculate the binomial probability of the number of customers who actually show up, if only 10 seats are sold. Enter these probabilities in the column marked  $P(X)$  in the table. Now, complete the part of the table for  $n = 10$ .

If no overbooking is done, on average how much money do they make on a plane that was sold out? (Remember, “no shows” get their money back.) \_\_\_\_\_

Calculate the binomial probability of the number of customers who actually show up if 11, 12, or 13 seats are sold. Enter these probabilities in the appropriate columns marked  $P(X)$  in the tables. Now complete the tables for  $n = 11$ ,  $n = 12$ , and  $n = 13$ .

If they overbook one reservation, on average how much revenue do they take in on a plane for which 11 tickets were sold? \_\_\_\_\_

If they overbook one reservation, on average what is their overbooking cost on a plane for which 11 tickets were sold? \_\_\_\_\_

If they overbook one reservation, on average how much profit do they make on a plane for which 11 tickets were sold? \_\_\_\_\_

Answer the analogous three questions for  $n = 12$  (overbook 2) and  $n = 13$  (overbook 3).

X	R(X)	C(X)	n = 10			n = 11					
			P(X)	R(X)*P(X)	C(X)*P(X)	P(X)	R(X)*P(X)	C(X)*P(X)			
0	0	0			0						
1	100	0			0						
2	200	0			0						
3	300	0			0						
4	400	0			0						
5	500	0			0						
6	600	0			0						
7	700	0			0						
8	800	0			0						
9	900	0			0						
10	1000	0			0						
11	1100	200	NA		0						
12	1200	400	NA		0	NA					
13	1300	600	NA		0	NA					
14	1400	800	NA		0	NA					
15	1500	1000	NA		0	NA					
$E(R(X)) = \sum R(X)*P(X) =$			$E(C(X)) =$			$E(R(X)) =$			$E(C(X)) =$		

X	R(X)	C(X)	n = 12			n = 13					
			P(X)	R(X)*P(X)	C(X)*P(X)	P(X)	R(X)*P(X)	C(X)*P(X)			
0	0	0									
1	100	0									
2	200	0									
3	300	0									
4	400	0									
5	500	0									
6	600	0									
7	700	0									
8	800	0									
9	900	0									
10	1000	0									
11	1100	200									
12	1200	400									
13	1300	600	NA								
14	1400	800	NA			NA					
15	1500	1000	NA			NA					
$E(R(X)) = \sum R(X)*P(X) =$			$E(C(X)) =$			$E(R(X)) =$			$E(C(X)) =$		



## Solutions to Extensions and Homework

### Extension 1

1. When *one* number cube is tossed, how many outcomes are possible? 6  
 When *two* are tossed?  $6^2=36$                       When *three* are tossed?  $6^3=216$

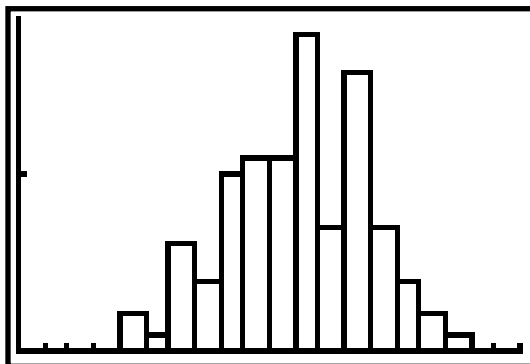
Now suppose we begin to think about what some of the outcomes for three number cubes look like.

2. Write one of the possible outcomes as an ordered triple  $(x, y, z)$ . *Answers vary*  
 Write another possible outcome in the same form. *Answers vary*  
 The entire probability space could be written in this way, if we could be assured of not missing any possibility. Can you think of a *systematic* way of writing all of the possible outcomes? *Answers may vary, but hopefully some students will see that one way to do so is to hold two of the numbers constant and let the third number vary across the six possibilities.*
3. How many ordered triples in the probability space have the form  $(1, 1, z)$ ? 6 List them all and next to each one, write the total of the three number cubes.  
 $(1,1,1), 3; (1,1,2), 4; (1,1,3), 5; (1,1,4), 6; (1,1,5), 7; (1,1,6), 8$
4. Repeat step 3 for all of the ordered triples of the form  $(1, 2, z)$ . Then do the same for ordered triples of the form  $(1, 3, z)$  through  $(6, 6, z)$ .  
 $(1,2,1), 4; (1,2,2), 5; (1,2,3), 6; (1,2,4), 7; (1,2,5), 8; (1,2,6), 9;$   
 $(1,3,1), 5; \dots; (1,3,6), 10; (1,4,1), 6; \dots; (1,4,6), 11; (1,5,1), 7; \dots; (1,5,6), 12;$   
 $(1,6,1), 8; \dots; (1,6,6), 13; (2,1,1), 4; \dots; (2,1,6), 9; (2,2,1), 5; \dots; (2,2,6), 10;$   
 $(2,3,1), 6; \dots; (2,3,6), 11; (2,4,1), 7; \dots; (2,4,6), 12; (2,5,1), 8; \dots; (2,5,6), 13;$   
 $(2,6,1), 9; \dots; (2,6,6), 14; \dots; (3,1,1), 5; \dots; (3,1,6), 10; (3,2,1), 6; \dots; (3,2,6), 11;$   
 $(3,3,1), 7; \dots; (3,3,6), 12; (3,4,1), 8; \dots; (3,4,6), 13; (3,5,1), 9; \dots; (3,5,6), 14;$   
 $(3,6,1), 10; \dots; (3,6,6), 15; (4,1,1), 6; \dots; (4,1,6), 11; (4,2,1), 7; \dots; (4,2,6), 12;$   
 $(4,3,1), 8; \dots; (4,3,6), 13; (4,4,1), 9; \dots; (4,4,6), 14; (4,5,1), 10; \dots; (4,5,6), 15;$   
 $(4,6,1), 11; \dots; (4,6,6), 16; (5,1,1), 7; \dots; (5,1,6), 12; (5,2,1), 8; \dots; (5,2,6), 13;$   
 $(5,3,1), 9; \dots; (5,3,6), 14; (5,4,1), 10; \dots; (5,4,6), 15; (5,5,1), 11; \dots; (5,5,6), 16;$   
 $(5,6,1), 12; \dots; (5,6,6), 17; (6,1,1), 8; \dots; (6,1,6), 13; (6,2,1), 9; \dots; (6,2,6), 14;$   
 $(6,3,1), 10; \dots; (6,3,6), 15; (6,4,1), 11; \dots; (6,4,6), 16; (6,5,1), 12; \dots; (6,5,6), 17$   
 $(6,6,1), 13; \dots; (6,6,6), 18$
5. For how many of the ordered pairs from steps 3 and 4 is the *total* of the number cubes 3? 1  
 6. For how many of the ordered pairs from steps 3 and 4 is the *total* 18? 1  
 7. What is the probability of obtaining a *total* of 3?  $1/216$  ... of 18?  $1/216$   
 8. In a similar manner, find the probabilities of obtaining *totals* of 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17.  $P(4)=3/216=P(17)$ ;  $P(5)=6/216=P(16)$ ;  $P(6)=10/216=P(15)$ ;  
 $P(7)=15/216=P(14)$ ;  $P(8)=21/216=P(13)$ ;  $P(9)=25/216=P(12)$ ;  $P(10)=27/216=P(11)$
9. Convert each of the probabilities from steps 7 and 8 to decimal form. How do these probabilities compare to the pooled relative frequencies from the Hot Dog Sales activity?  
 $1/216=.005$ ;  $3/216=.014$ ;  $6/216=.028$ ;  $10/216=.046$ ;  $15/216=.069$ ;  $21/216=.097$ ;

$25/216=.116$ ;  $27/216=.125$ ; Comparisons will vary

### EXTENSION 2:

1. A typical histogram generated by the “NEWSBOY” program is shown below. Students should observe that the numbers in the middle (e.g., 8, 9, 10, 11, 12, 13) generally have higher frequencies than the numbers on the low and high extremes.



2. 500
3. In general, the patterns noticed in the first histogram should be more pronounced in the second. There should be fewer “spikes”, and the histogram may begin to hint at a bell-shaped normal distribution.
4. As the number of rolls of the number cubes increases, the histogram should smooth out and begin to take on more characteristics of a normal distribution. This is the Law of Large Numbers at work.

### EXTENSION 3:

1. The graph is perfectly symmetric and the middle numbers have much larger frequencies than the numbers on either extreme. Some students might notice that the histogram appears to have the same shape as a normal distribution.
2. None of the histograms from the Student Activity or the previous Extension has perfect symmetry.
3. Graphing  $Y1=\text{normalPdf}(X-0.5,10.5,2.96)$  produces a normal curve with mean 10.5 and standard deviation 2.96. Students should recognize the bell-shaped curve.  $X-0.5$  was used rather than  $X$ , in order to effect a horizontal shift of one-half unit. This was necessary so that the normal curve would be aligned with the histogram, because “normalPdf(” is a continuous approximation to a discrete distribution.
4.  $Y1$  is a good fit with the histogram.

**Homework**

**1. Salvage Value**

The unsold hot dogs and buns will be sold at a steep discount to fans as they leave. Assume that you will be able to sell all of your leftover dozens at a price of \$5 per dozen. Use the tables below to reevaluate the policy of ordering 11, 12, or 13 dozen with this “salvage” value of \$5 per dozen. For the column “Pooled Relative Frequency” use the data generated earlier in class. For example, if you ordered 11 dozen and the demand was only nine dozen, there would be two dozen to sell at a discount. The total net revenue would increase by  $2(\$5) = \$10$ .

After completing the tables below discuss the affect of salvage value on each ordering policy.

What was the best policy with no salvage value? *The answers will vary depending upon the results of your pooled relative frequency. We used the actual probabilities and found the optimal was to order 12 with an expected value of \$141.96 per week.*

What was the best policy with a \$5 salvage value? *The answers will vary depending upon the results of your pooled relative frequency. We used the actual probabilities and found the optimal was to order 13 with an expected value of \$154.31 per week.*

*The table below represents the expected values based on probabilities. Your answers should be in the “range” of these values but due to randomness, the optimal strategy may differ.*

	Order 11	Order 12	Order 13
Salvage \$0	\$140.96	\$141.96	\$140.18
Salvage \$5	\$148.26	\$152.39	\$154.31

Always Order <u>11</u> dozen									
Demand	Hot dogs sold	No Salvage Value		\$5/dozen Salvage Value		Pooled Relative Freq.	No Salvage Value	\$5/dozen Salvage Value	
		Total revenue	Profit A	Total revenue	Profit B		Profit A X Pooled Relative Frequency	Profit B X Pooled Relative Frequency	
3	3	72	-16	112	24				
4	4	96	8	131	43				
5	5	120	32	150	62				
6	6	144	56	169	81				
7	7	168	80	188	100				
8	8	192	104	207	119				
9	9	216	128	226	138				
10	10	240	152	245	157				
11	11	264	176	264	176				
12	11	264	176	264	176				
13	11	264	176	264	176				
14	11	264	176	264	176				
15	11	264	176	264	176				
16	11	264	176	264	176				
17	11	264	176	264	176				
18	11	264	176	264	176				
<b>Weighted Average = (Each Column Sum)</b>									

Note: Answers for the three remaining blank columns in the tables will depend on the pooled relative frequencies your students obtained in class when they completed the Student Activity.

<b>Always Order <u>12</u> dozen</b>									
<i>Demand</i>	<i>Hot dogs sold</i>	<i>No Salvage Value</i>		<i>\$/dozen Salvage Value</i>		<i>Pooled Relative Freq.</i>	<i>No Salvage Value</i>	<i>\$/dozen Salvage Value</i>	
		<i>Total revenue</i>	<i>Profit A</i>	<i>Total revenue</i>	<i>Profit B</i>		<i>Profit A X Pooled Relative Frequency</i>	<i>Profit B X Pooled Relative Frequency</i>	
3	3	72	-24	117	21				
4	4	96	0	136	40				
5	5	120	24	155	59				
6	6	144	48	174	78				
7	7	168	72	193	97				
8	8	192	96	212	116				
9	9	216	120	231	135				
10	10	240	144	250	154				
11	11	264	168	269	173				
12	12	288	192	288	192				
13	12	288	192	288	192				
14	12	288	192	288	192				
15	12	288	192	288	192				
16	12	288	192	288	192				
17	12	288	192	288	192				
18	12	288	192	288	192				
<b>Weighted Average = (Each Column Sum)</b>									

<b>Always Order <u>13</u> dozen</b>									
<i>Demand</i>	<i>Hot dogs sold</i>	<i>No Salvage Value</i>		<i>\$/dozen Salvage Value</i>		<i>Pooled Relative Freq.</i>	<i>No Salvage Value</i>	<i>\$/dozen Salvage Value</i>	
		<i>Total revenue</i>	<i>Profit A</i>	<i>Total revenue</i>	<i>Profit B</i>		<i>Profit A X Pooled Relative Frequency</i>	<i>Profit B X Pooled Relative Frequency</i>	
3	3	72	-32	122	18				
4	4	96	-8	141	37				
5	5	120	16	160	56				
6	6	144	40	179	75				
7	7	168	64	198	94				
8	8	192	88	217	113				
9	9	216	112	236	132				
10	10	240	136	255	151				
11	11	264	160	274	170				
12	12	288	184	293	189				
13	13	312	208	312	208				
14	13	312	208	312	208				
15	13	312	208	312	208				
16	13	312	208	312	208				
17	13	312	208	312	208				
18	13	312	208	312	208				
<b>Weighted Average = (Each Column Sum)</b>									

## 2. Airline Overbooking

Explain why “NA” appears in specific boxes below. *Those values cannot occur. For example, eleven people cannot show up with confirmed reservations, if the policy limited the number of reservations to ten.*

Explain why  $E(C(X))$  for  $n = 10$  is always zero. *Since no overbooking occurs when  $n=10$  there can be no cost of overbooking.*

Calculate the binomial probability of the number of customers who actually show up if only 10 seats are sold. Enter these probabilities in the column marked  $P(X)$  in the table. Now complete the part of the table for  $n = 10$ .

*Note to teacher: You may choose to provide these probabilities to the students rather than have them calculate them.*

If no overbooking is done, on average how much money do they make on a plane that was sold out? (Remember, “no shows” get their money back.) \$800

Calculate the binomial probability of the number of customers who actually show up if 11,12, or 13 seats are sold. Enter these probabilities in the appropriate columns marked  $P(X)$  in the tables. Now complete the tables for  $n = 11$ ,  $n = 12$ , and  $n = 13$ . *The completed tables appear on the following pages.*

If they overbook one reservation, on average how much revenue do they take in on a plane for which 11 tickets were sold? \$880

If they overbook one reservation, on average what is their overbooking cost on a plane for which 11 tickets were sold? \$17.18

If they overbook one reservation, on average how much profit do they make on a plane for which 11 tickets were sold?  $\$880 - \$17.18 = \$862.82$

Answer the analogous three questions for  $n = 12$  (overbook 2) and  $n = 13$  (overbook 3).

$n = 12$ : \$960, \$68.72, \$891.28

$n = 13$ : \$1040, \$158.05, \$881.95

			$n = 10$			$n = 11$		
$X$	$R(X)$	$C(X)$	$P(X)$	$E(R(X)) = R(X)*P(X)$	$E(C(X)) = C(X)*P(X)$	$P(X)$	$E(R(X)) = R(X)*P(X)$	$E(C(X)) = C(X)*P(X)$
0	0	0	0.000	\$0.00	\$0.00	0.000	\$0.00	\$0.00
1	100	0	0.000	\$0.00	\$0.00	0.000	\$0.00	\$0.00
2	200	0	0.000	\$0.01	\$0.00	0.000	\$0.00	\$0.00
3	300	0	0.001	\$0.24	\$0.00	0.000	\$0.06	\$0.00
4	400	0	0.006	\$2.20	\$0.00	0.002	\$0.69	\$0.00
5	500	0	0.026	\$13.21	\$0.00	0.010	\$4.84	\$0.00
6	600	0	0.088	\$52.85	\$0.00	0.039	\$23.25	\$0.00
7	700	0	0.201	\$140.93	\$0.00	0.111	\$77.51	\$0.00
8	800	0	0.302	\$241.59	\$0.00	0.221	\$177.17	\$0.00
9	900	0	0.268	\$241.59	\$0.00	0.295	\$265.75	\$0.00
10	1000	0	0.107	\$107.37	\$0.00	0.236	\$236.22	\$0.00
11	1100	200	NA			0.086	\$94.49	\$17.18
12	1200	400	NA			NA		
13	1300	600	NA			NA		
14	1400	800	NA			NA		
15	1500	1000	NA			NA		
$E(R(X)) = \sum R(X)*P(X) =$				\$800	$E(C(X)) = \$0.00$		$E(R(X)) = \$880$	$E(C(X)) = \$17.18$
				NET = \$800			NET = \$862.82	

			$n = 12$			$n = 13$		
$X$	$R(X)$	$C(X)$	$P(X)$	$E(R(X)) = R(X)*P(X)$	$E(C(X)) = C(X)*P(X)$	$P(X)$	$E(R(X)) = R(X)*P(X)$	$E(C(X)) = C(X)*P(X)$
0	0	0	0.000	\$0.00	\$0.00	0.000	\$0.00	\$0.00
1	100	0	0.000	\$0.00	\$0.00	0.000	\$0.00	\$0.00
2	200	0	0.000	\$0.00	\$0.00	0.000	\$0.00	\$0.00
3	300	0	0.000	\$0.02	\$0.00	0.000	\$0.00	\$0.00
4	400	0	0.001	\$0.21	\$0.00	0.000	\$0.06	\$0.00
5	500	0	0.003	\$1.66	\$0.00	0.001	\$0.54	\$0.00
6	600	0	0.016	\$9.30	\$0.00	0.006	\$3.45	\$0.00
7	700	0	0.053	\$37.21	\$0.00	0.023	\$16.12	\$0.00
8	800	0	0.133	\$106.30	\$0.00	0.069	\$55.28	\$0.00
9	900	0	0.236	\$212.60	\$0.00	0.154	\$138.19	\$0.00
10	1000	0	0.283	\$283.47	\$0.00	0.246	\$245.67	\$0.00
11	1100	200	0.206	\$226.77	\$41.23	0.268	\$294.81	\$53.60
12	1200	400	0.069	\$82.46	\$27.49	0.179	\$214.40	\$71.47
13	1300	600	NA			0.055	\$71.47	\$32.99
14	1400	800	NA			NA		
15	1500	1000	NA			NA		
$E(R(X)) = \sum R(X)*P(X) =$				\$960	$E(C(X)) = \$68.72$		$E(R(X)) = \$1,040$	$E(C(X)) = \$158.05$
				NET = \$891.28			NET = \$881.95	