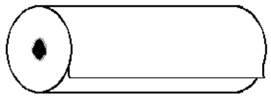
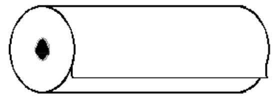
Trimming Loss at the *Cutting Times*

The *Cutting Times* is a small local newspaper. The paper receives large 48-inch wide rolls of newsprint, which must be cut into 25-inch wide rolls for the ordinary pages in the paper and 21-inch wide rolls for smaller inserts.



Pattern A



Pattern B

Using the objects above, draw a model of each possible cutting pattern. Label each roll with its dimensions. Be sure to label any waste on each roll.

1 Describe the two most efficient patterns which may be used to cut the large rolls of newsprint, so as to produce the needed smaller rolls. Be sure to include the number of inches of newsprint wasted for each pattern.

	# of 25 inch rolls	# of 21 inch rolls	inches of paper wasted
Pattern A			
Pattern B			

For the Sunday edition of the *Times* right before Thanksgiving, the paper needs twenty 25-inch rolls for the ordinary part of the edition, and fifty 21-inch rolls for all of the extra advertising inserts.

2 Recommend how many of each pattern should be cut. ____

Compare your recommendation with others near you. Ask why they made their recommendation.

3 When the newspaper decides how many of each pattern to cut, what should they consider?

____

____

____

4 How can the newspaper control the amount of waste?

____

5 Why would the newspaper want to control the amount of waste?

____

Decision Variables:

An important step in the solution of this problem is identifying the objective and the variables which can be controlled. The aspects of a problem situation which determine the objective and which can be controlled by a manager are called *decision variables*. Although it does not matter how the patterns are named, for uniformity we will define the decision variables in the *Cutting Times* problem as:

x = the number of large rolls cut into two 21-inch rolls;

y = the number of large rolls cut into one 25-inch and one 21-inch roll.

Decision variables:
A set of variable quantities completely describing the decisions to be made.

Label your patterns in number 1 with x and y.

Objective Functions:

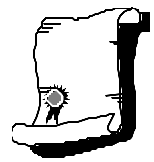
6 Represent the amount of waste, w, using the decision variables; that is, in terms of x and y?

____

Is the Cutting Times interested in minimizing or maximizing waste? ______________________________

Since w is a function of x and y and represents the newspaper's objective, the function you have just written is called the *objective function*.

Objective Function:
A quantity to be maximized or minimized which is defined in terms of the decision variables.



Constraints:

7 How many 25-inch rolls and how many 21-inch rolls are produced when a large roll is cut into two 21-inch rolls?

_________________ = the number of 25-inch rolls; _________________ = the number of 21-inch rolls

8 How many 25-inch rolls and how many 21-inch rolls are produced when a large roll is cut into one 25-inch roll and one 21-inch roll?

_________________ = the number of 25-inch rolls; _________________ = the number of 21-inch rolls

9 If 5 rolls are cut using the pattern described in number 7, and 10 rolls are cut using the pattern in number 8, how many 25-inch rolls and how many 21-inch rolls will be produced?

_________________ = the number of 25-inch rolls; _________________ = the number of 21-inch rolls

10 If x rolls are cut using the pattern described in number 7, and y rolls are cut using the pattern in number 8, how many 25-inch rolls and how many 21-inch rolls will be produced?

_________________ = the number of 25-inch rolls; _________________ = the number of 21-inch rolls

11 Use the problem requirements at the end of number 1 to define two inequalities based on the number of 25-inch and 21-inch rolls produced.

25-inch roll requirement: ____________________; 21-inch roll requirement:______________________

Each of the inequalities affects, or constrains, the values which the decision variables may take, so the inequalities you have just written are called *constraints*.

Constraints:
Restrictions on the values of one or more of the decision variables.

You have just completed the formulation of a linear programming problem. Algebraically, we can state the problem as:

Minimize $w = 6x + 2y$, subject to $2x + y \geq 50$ and $y \geq 20$.

Identifying What the Solution Could Be – Locating the Feasible Region

One way to investigate all of the possible problem solutions is to graph the system of constraint inequalities. Graph the constraint inequalities using graph paper or a graphing utility. Notice that one of the inequalities restricts the possible values of only one of the decision variables.

12 Which decision variable is this? ____

In terms of the problem, why is this variable restricted in this way? ____________________________

____

Feasible Region:
The set of all points which satisfy all of the constraints.

On your graph, plot the point that represents your recommendation from number 2.

13 Is your recommendation a possible solution? ___________

Shade the region of the plane that represents all possible solutions. The region you have just shaded, which contains all the possible, or feasible, solutions, is called the *feasible region*. The best solution to the problem must lie somewhere in the feasible region.

The Search for Optimality - Finding the Best Solution:

In order to identify the best possible solution, next you will investigate the amount of waste produced by given solutions. Choose two points in the feasible region and plot them on the graph of the feasible region.

14 In terms of the original problem, discuss what these points represent. ___________________________

____

____

15 For each of the points that you selected, use the objective function to calculate the amount of waste produced for that point.

w_1 =___________________________ w_2 =______________________________

Graph the equations $w_1 = 6x + 2y$ and $w_2 = 6x + 2y$ on your graph of the feasible region.

16 What do you notice about these two graphs?

____

17 With respect to one another, how can you describe the location of the line which represents the smaller amount of waste.

____

On your graph of the feasible region, draw the line representing the least possible waste while still intersecting the feasible region.

18 At what point does this line intersect the feasible region? ___________

19 How would you describe the location of this point in the feasible region?

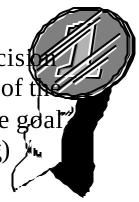
____

20 What do the coordinates mean in terms of the original problem?

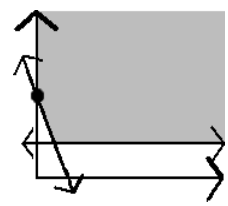
____

The best possible solution in a linear programming problem is called the *optimal solution*. If there is a unique solution to a linear programming problem, it must occur at one of the *corner points* of the feasible region.

The Optimal Solution:
The set of values of the decision variables which satisfies all of the constraints and achieves the goal of minimizing (or maximizing) the objective function.



The Corner Principle:
If there is a unique solution to a linear programming problem, it must occur at one of the corner points of the feasible region.



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