Trimming Loss at the *Cutting Times*

Extension 1: What if any extra 25-inch rolls cut in solving the *Cutting Times* problem cannot be used in the future and must be treated as waste? How would this change the problem formulation and solution?

Extension 2: Suppose the *Cutting Times* can purchase large 72-inch wide rolls of newsprint.

a) List the possible patterns that can be cut.

b) How would this change affect the problem formulation?

c) How would this change affect the problem solution?

d) Formulate the new problem by finding the objective function and the system of constraints.

e) How would such a problem be solved in the world of business and industry?

Extension 3: Suppose the large rolls to be cut are 75 inches wide.

a) List the possible cutting patterns.

b) Write the objective function expressing the total waste.

c) Write the system of constraints.

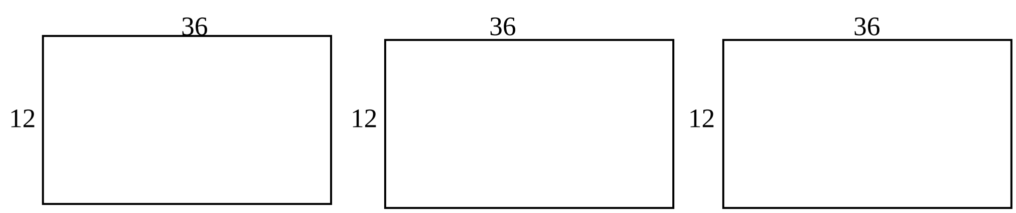
Homework

Problem 1: A health care corporation wants to cut large 55-inch wide beams of gauze into 5-inch and 4-inch widths. List all of the possible cutting patterns.

Problem 2: What variable names could be assigned to represent the number of beams of gauze cut using each of the patterns?

Problem 3: The company wants to further cut some of the 5-inch rolls of gauze into 3/4-inch rolls to be used in the manufacture of adhesive strips and 3-inch rolls to be used in the manufacture of square gauze pads. The company needs 120,000 3/4-inch rolls and 10,000 3-inch rolls. How shall the 5-inch rolls be cut so as to minimize waste?

Problem 4: Mrs. Miller has 3 sheets of 36" by 24" poster board that she wants to cut into 12" by 9" and 5" by 6 " pieces. Her 16 students are to sketch student portraits on the larger sheets and silhouettes on the smaller sheets. Find at least one pattern for cutting the sheets that would yield the at least one large piece of waste that could be used in another project. Make a drawing of each of the patterns. Write a paragraph that supports your solution and the rational that this would be the optimal solution.



The number of each of the two patterns that could be cut from each of the three sheets is:

Problem 5: The Mathfield Meat Industry cuts beef carcasses into two different patterns to provide for the varying levels of meat quality. One pattern of cutting utilizes the premium meat sections costing $20 to cut and the second pattern cost $12. Due to the length of time it takes to cut pattern one, the company decides to cut no more than 1000 carcasses per week using this pattern. There is also a demand for no more than 1800 carcasses to be cut each week. How many of each pattern should Mathfield cut in order to minimize cost of cutting the beef carcasses?

1. Identify the decision variables.

2. Use the decision variables to define an objective function.

3. Write any constraints on the decision variables as inequalities.

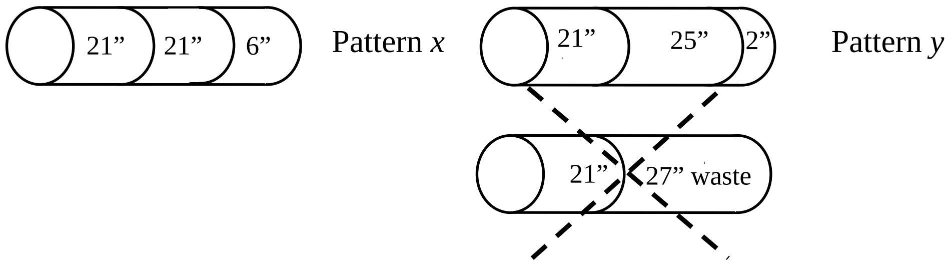
4. Graph the feasible region.

5. Locate the corner points of the feasible region, find their coordinates, and determine the optimal solution. What does the optimal solution mean in terms of the given information in the problem?

6. How does this problem differ from a trim-loss linear programming problem?

Solutions to Extensions

Extension 1



If any extra 25-inch rolls are waste, then any more than 20 large rolls cut using pattern *y* actually produces 27 inches of waste. Therefore it would not be logical to cut more than 20 rolls using pattern *y*.

If 20 rolls are cut using pattern *y*, that produces 20 25-inch rolls and 20 21-inch rolls. The requirement for 25-inch rolls has been met and we need 30 additional 21-inch rolls. Cutting 15 large rolls using pattern *x* can most efficiently produce these.

So in this case, the other corner point (15, 20) is now the optimal solution.

Because the unneeded rolls are now considered waste, the formulation would need to be changed as follows:

e_1 = excess rolls of 21" width
e_2 = excess rolls of 25" width

Minimize $w = 6x + 2y + 21e_1 + 25e_2$ subject to: $2x + y - e_1 = 50$ and $y - e_2 = 20$

Extension 2

a)

Pattern	**Length of Cut**	**Length of Cut**	**Length of Cut**	**Waste**
x	25”	25”	21”	1”
y	25”	21”	21”	5”
z	21”	21”	21”	9”

b) There are now three patterns, therefore there will be 3 decision variables.

c) The 3 decision variables place this problem situation in 3-space. At this point you may want to consider posing questions regarding the 3-dimensional nature of this problem. e.g. What does the graph of $y = 20$ look like in 3-dimensions? (You may want to pose the same question for $2x + y = 50$. What is the analog of a 2-dimensional line in one-dimension [a point] or in 3-dimensions [a plane]?)

d) minimize: $w = x + 5y + 9z$
subject to: $2x + y \geq 20$ (25-inch constraint)
$x + 2y + 3z \geq 50$ (21-inch constraint)

e) Because it is difficult to visualize the feasible region, a computer program would most likely be used to analyze this problem.

<u>Extension 3</u>

a)

Pattern	**Length of Cut**	**Length of Cut**	**Length of Cut**	**Waste**
A	25”	25”	25”	0”
B	25”	25”	21”	4”
C	25”	21”	21”	8”
D	21”	21”	21”	12”

b) minimize: $w = 4b + 8c + 12d$

c) subject to: $3a + 2b + c \geq 20$ (25-inch constraint)
$b + 2c + 3d \geq 50$ (21-inch constraint)

n.b. You may want to point out to students that the solution to this problem would have to be done using a computer program. It is impossible to visualize a four-dimensional feasible region and therefore these numerical computations would be more efficiently analyzed by a computer program.

<u>Homework Answers:</u>

<u>Problem 1:</u>

5” strips	4” strips	waste
11	0	0
10	1	1
9	2	2
8	3	3
7	5	0
6	6	1
5	7	2
4	8	3
3	10	0
2	11	1
1	12	2
0	13	3

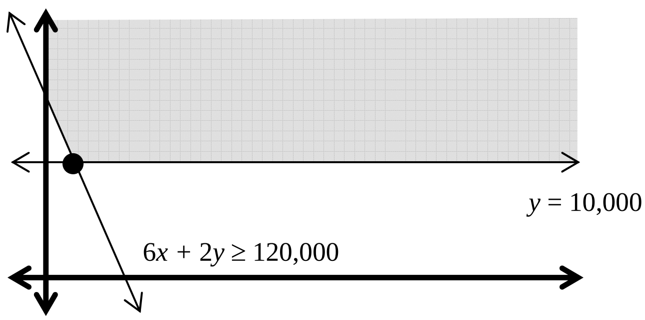
Problem 2:

Use the first 12 letters of the alphabet. A better solution would be the use of subscripts.
i.e. $x_1, x_2, x_3 \ldots x_{10}, x_{11}, x_{12}$.

Problem 3:

Let x = the number of 5” strips cut into 6 ¾” rolls
Let y = the number of 5” strips cut into 1 3” roll and 2 ¾” rolls

Minimize w = 6x + 2y subject to the constraints 2x + y ≥ 120,000 and y ≥ 10,000



16,667 = x
10,000 = y

Problem 4:

	1	2	3
12 x 9	6	6	4
5 x 6	6	6	4
waste	3 x 6	3 x 6	12 x 26

	1	2	3
12 x 9	8	8	0
5 x 6	0	0	16
waste	0	0	4 x 10

	1	2	3
12 x 9	5	5	6
5 x 6	8	8	0
waste	12 x 7	12 x 7	12 x 18

Above are three possible solutions to the homework. These are by no means the only possible solutions, and it will be necessary to judge individually the various rationales of the students’ solutions.

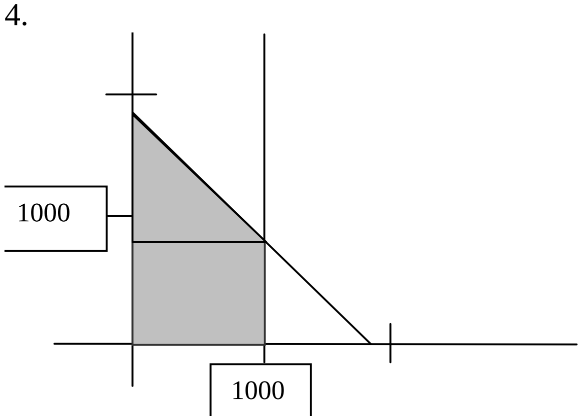
Problem 5:

1. Let: x = the number of carcasses cut using the pattern costing \$20
   y = the number of carcasses cut using the pattern costing \$12.

2. If c = the total cost per week of cutting the carcasses, then $c = 20x + 12y$.

3. The constraints are: $x \leq 1000$ and $x + y \leq 1800$.

4.



5. The corner points are (0,0), (0,1000), (1000,800), and (1000,0). Using the corner principle and assuming that the company wants to cut *some* carcasses (thus eliminating the corner point at the origin), the optimal solution occurs at (1000,0); i.e., cut 1000 carcasses according to the more expensive pattern and none according to the other pattern.

6. It was the cost of cutting a pattern, rather than the product output from the pattern, that determined the objective function. This problem actually is more similar to a product mix problem, such as that developed in our *High Step Shoes* module.